

## 5. Bearing internal load distribution and displacement

### 5.1 Bearing internal load distribution

This section will begin by examining the effect of a radial load  $F_r$  and an axial load  $F_a$  applied on a single-row bearing with a contact angle  $\alpha$  (angular contact ball bearings, tapered roller bearings, etc.). The ratio of  $F_a$  to  $F_r$  determines the range of the loading area when just a portion of the raceway sustains the load, or when the entire raceway circumference sustains the load.

The size of the loading area is called the load factor  $\epsilon$ . When only a part of the outer circumference bears the load,  $\epsilon$  is the ratio between the projected length of the loading area and the raceway diameter. For this example, we use  $\epsilon \leq 1$ . (Refer to Fig. 1).

When the entire raceway circumference is subjected to a load, the calculation becomes,

$$\epsilon = \frac{\delta_{\max}}{\delta_{\max} - \delta_{\min}} \geq 1$$

where,  $\delta_{\max}$ : Elastic deformation of a rolling element under maximum load

$\delta_{\min}$ : Elastic deformation of a rolling element under minimum load

The load  $Q(\psi)$  on any random rolling element is proportional to the amount of elastic deformation  $\delta(\psi)$  of the contact surface raised to the  $t$  power. Therefore, it follows that when  $\psi=0$  (with maximum rolling element load of  $Q_{\max}$  and maximum elastic deformation of  $\delta_{\max}$ ),

$$\frac{Q(\psi)}{Q_{\max}} = \left( \frac{\delta(\psi)}{\delta_{\max}} \right)^t \dots\dots\dots (1)$$

$t=1.5$  (point contact),  $t=1.1$  (line contact)

The relations between the maximum rolling element load  $Q_{\max}$ , radial load  $F_r$ , and axial load  $F_a$  are as follows:

$$F_r = J_r Z Q_{\max} \cos \alpha \dots\dots\dots (2)$$

$$F_a = J_a Z Q_{\max} \sin \alpha \dots\dots\dots (3)$$

where  $Z$  is the number of rolling elements, and  $J_r$  and  $J_a$  are coefficients for point and line contact derived from Equation (1). The values for  $J_r$  and  $J_a$  with corresponding  $\epsilon$  values are

given in Table 1. When  $\epsilon=0.5$ , (when half of the raceway circumference is subjected to a load), the relationship between  $F_a$  and  $F_r$  becomes,

$$F_a = 1.216 F_r \tan \alpha \dots\dots\dots (\text{point contact})$$

$$F_a = 1.260 F_r \tan \alpha \dots\dots\dots (\text{line contact})$$

The basic load rating of radial bearings becomes significant under these conditions.

Assuming the internal clearance in a bearing  $\Delta=0$ ,  $\epsilon=0.5$  and using a value for  $J_r$  taken from Table 1, Equation (2) becomes,

$$Q_{\max} = 4.37 \frac{F_r}{Z \cos \alpha} \text{ point contact} \dots\dots\dots (4)$$

$$Q_{\max} = 4.08 \frac{F_r}{Z \cos \alpha} \text{ line contact} \dots\dots\dots (5)$$

With a pure axial load,  $F_r=0$ ,  $\epsilon=\infty$ ,  $J_a=1$ , and Equation (3) becomes;

$$Q = Q_{\max} = \frac{F_a}{Z \sin \alpha} \dots\dots\dots (6)$$

(In this case, the rolling elements all share the load equally.)

For a single-row deep groove ball bearing with zero clearance that is subjected to a pure radial load, the equation becomes;

$$Q_{\max} = 4.37 \frac{F_r}{Z} \dots\dots\dots (7)$$

For a bearing with a clearance  $\Delta > 0$  subjected to a radial load and with  $\epsilon < 0.5$ , the maximum rolling element load will be greater than that given by Equation (7). Also, if the outer ring is mounted with a clearance fit, the outer ring deformation will reduce the load range. Equation (8) is a more practical relation than Equation (7), since bearings usually operate with some internal clearance.

$$Q_{\max} = 5 \frac{F_r}{Z} \dots\dots\dots (8)$$

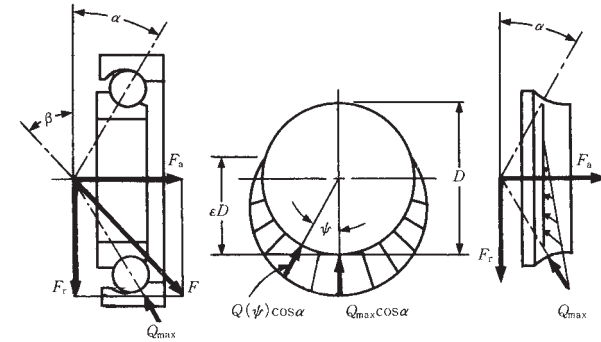


Fig. 1

Table 1 Values for  $J_r$  and  $J_a$  in single-row bearings

| $\epsilon$ | Point contact                 |        |        | Line contact                  |        |        |
|------------|-------------------------------|--------|--------|-------------------------------|--------|--------|
|            | $\frac{F_r \tan \alpha}{F_a}$ | $J_r$  | $J_a$  | $\frac{F_r \tan \alpha}{F_a}$ | $J_r$  | $J_a$  |
| 0          | 1                             | 0      | 0      | 1                             | 0      | 0      |
| 0.1        | 0.9663                        | 0.1156 | 0.1196 | 0.9613                        | 0.1268 | 0.1319 |
| 0.2        | 0.9318                        | 0.1590 | 0.1707 | 0.9215                        | 0.1737 | 0.1885 |
| 0.3        | 0.8964                        | 0.1892 | 0.2110 | 0.8805                        | 0.2055 | 0.2334 |
| 0.4        | 0.8601                        | 0.2117 | 0.2462 | 0.8380                        | 0.2286 | 0.2728 |
| 0.5        | 0.8225                        | 0.2288 | 0.2782 | 0.7939                        | 0.2453 | 0.3090 |
| 0.6        | 0.7835                        | 0.2416 | 0.3084 | 0.7480                        | 0.2568 | 0.3433 |
| 0.7        | 0.7427                        | 0.2505 | 0.3374 | 0.6999                        | 0.2636 | 0.3766 |
| 0.8        | 0.6995                        | 0.2559 | 0.3658 | 0.6486                        | 0.2658 | 0.4098 |
| 0.9        | 0.6529                        | 0.2576 | 0.3945 | 0.5920                        | 0.2628 | 0.4439 |
| 1.0        | 0.6000                        | 0.2546 | 0.4244 | 0.5238                        | 0.2523 | 0.4817 |
| 1.25       | 0.4338                        | 0.2289 | 0.5044 | 0.3598                        | 0.2078 | 0.5775 |
| 1.67       | 0.3088                        | 0.1871 | 0.6060 | 0.2340                        | 0.1589 | 0.6790 |
| 2.5        | 0.1850                        | 0.1339 | 0.7240 | 0.1372                        | 0.1075 | 0.7837 |
| 5          | 0.0831                        | 0.0711 | 0.8558 | 0.0611                        | 0.0544 | 0.8909 |
| $\infty$   | 0                             | 0      | 1      | 0                             | 0      | 1      |

### 5.2 Radial clearance and load factor for radial ball bearings

The load distribution will differ if there is some radial clearance. If any load acts on a bearing, in order for the inner and outer rings to maintain parallel rotation, the inner and outer rings must move relative to each other out of their original unloaded position. Movement in the axial direction is symbolized by  $\delta_a$  and that in the radial direction by  $\delta_r$ . With a radial clearance  $\Delta_r$  and a contact angle  $\alpha$ , as shown in Fig. 1, the total elastic deformation  $\delta(\psi)$  of a rolling element at the angle  $\psi$  is given by Equation (1).

$$\delta(\psi) = \delta_r \cos\psi \cos\alpha + \delta_a \sin\alpha - \frac{\Delta_r}{2} \cos\alpha \quad \dots\dots\dots (1)$$

The maximum displacement  $\delta_{max}$  with  $\psi=0$  is given,

$$\delta_{max} = \delta_r \cos\psi + \delta_a \sin\alpha - \frac{\Delta_r}{2} \cos\alpha \quad \dots\dots\dots (2)$$

Combining these two equations,

$$\delta(\psi) = \delta_{max} \left\{ 1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right\} \quad \dots\dots\dots (3)$$

and,

$$\varepsilon = \frac{\delta_{max}}{2\delta_r \cos\alpha} = \frac{1}{2} \left\{ 1 + \frac{\delta_a}{\delta_r} \tan\alpha - \frac{\Delta_r}{2\delta_r} \right\} \quad \dots\dots\dots (4)$$

When there is no relative movement in the axial direction, ( $\delta_a=0$ ), Equations (2) and (4) become,

$$\delta_{max} = \left( \delta_r - \frac{\Delta_r}{2} \right) \cos\alpha \quad \dots\dots\dots (2)'$$

$$\varepsilon = \frac{1}{2} \left( 1 - \frac{\Delta_r}{2\delta_r} \right) \quad \dots\dots\dots (4)'$$

$$\therefore \delta_{max} = \frac{\varepsilon}{1 - \varepsilon} \Delta_r \cos\alpha \quad \dots\dots\dots (5)$$

From the Hertz equation,

$$\delta_{max} = C \frac{Q_{max}^{2/3}}{D_w^{1/3}} \quad \dots\dots\dots (6)$$

The maximum rolling element load  $Q_{max}$  is given by,

$$Q_{max} = \frac{F_r}{J_r Z \cos\alpha} \quad \dots\dots\dots (7)$$

Combining Equations (5), (6), and (7) yields Equation (8) which shows the relation among radial clearance, radial load, and load factor.

$$\Delta_r = \left( \frac{1 - 2\varepsilon}{\varepsilon} J_r^{-2/3} \right) c \left( \frac{F_r}{Z} \right)^{2/3} D_w^{-1/3} \cos^{-5/3} \alpha \quad \dots\dots\dots (8)$$

- where,  $\Delta_r$ : Radial clearance (mm)
- $\varepsilon$ : Load factor
- $J_r$ : Radial integral (Page 111, Table 1)
- $c$ : Hertz elasticity coefficient
- $F_r$ : Radial load (N), {kgf}
- $Z$ : Number of balls
- $D_w$ : Ball diameter (mm)
- $\alpha$ : Contact angle ( $^\circ$ )

Values obtained using Equation (8) for a 6208 single-row radial ball bearing are plotted in Fig. 2.

As an example of how to use this graph, assume a radial clearance of 20  $\mu\text{m}$  and  $F_r = C_r/10 = 2\,910\text{ N}$  {297 kgf}. The load factor  $\varepsilon$  is found to be 0.36 from Fig. 2 and  $J_r = 0.203$  (Page 111, Table 1). The maximum rolling element load  $Q_{max}$  can then be calculated as follows,

$$Q_{max} = \frac{F_r}{J_r Z \cos\alpha} = \frac{2\,910}{0.203 \times 9} = 1\,590\text{ N} \text{ (163 kgf)}$$

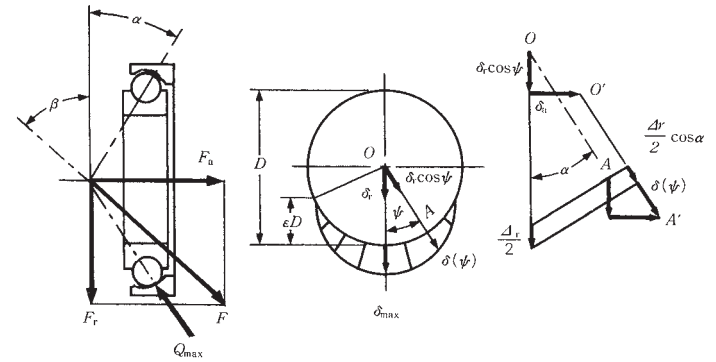


Fig. 1

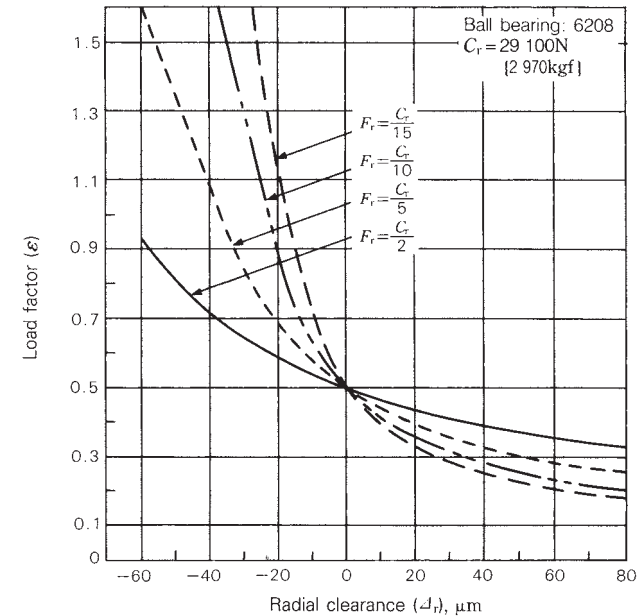


Fig. 2

**5.3 Radial clearance and maximum rolling element load**

If we consider an example where a deep groove ball bearing is subjected to a radial load and the radial clearance  $\Delta_r$  is 0, then the load factor  $\epsilon$  will be 0.5. When  $\Delta_r > 0$  (where there is a clearance),  $\epsilon < 0.5$ ; when  $\Delta_r < 0$ ,  $\epsilon > 0.5$ . (See Fig. 1).

Fig. 2 in section 5.2 shows how the load factor change due to clearance decreases with increasing radial load.

When the relationship between radial clearance and load factor is determined, it can be used to establish the relationship between radial clearance and bearing life, and between radial clearance and maximum rolling element load.

The maximum rolling element load is calculated using Equation (1).

$$Q_{\max} = \frac{F_r}{J_r Z \cos \alpha} \dots \dots \dots (1)$$

where,  $F_r$ : Radial load (N), {kgf}  
 $J_r$ : Radial integral  
 $Z$ : Number of balls  
 $\alpha$ : Contact angle (°)

$J_r$  is dependent on the value of  $\epsilon$  (Page 111, Table 1), and  $\epsilon$  is determined, as explained in Section 5.2, from the radial load and radial clearance.

Fig. 2 shows the relationship between the radial clearance and maximum rolling element load for a 6208 deep groove ball bearing. As can be seen from Fig. 2, the maximum rolling element load increases with increasing radial clearance or reduction in the loaded range. When the radial clearance falls slightly below zero, the loaded range grows widely resulting in minimum in the maximum rolling element load. However, as the compression load on all rolling elements is increased when the clearance is further reduced, the maximum rolling element load begins to increase sharply.

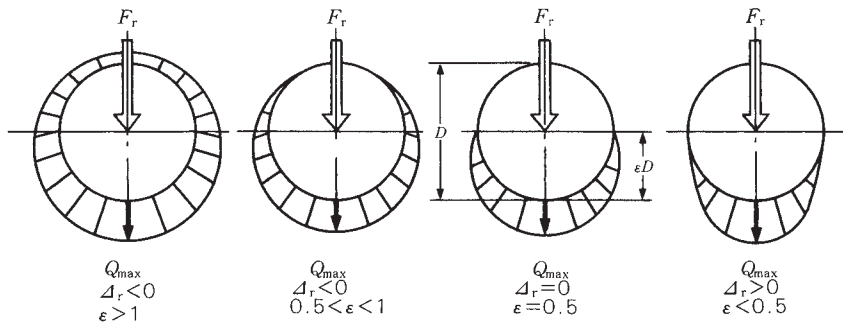


Fig. 1 Radial clearance and load distribution

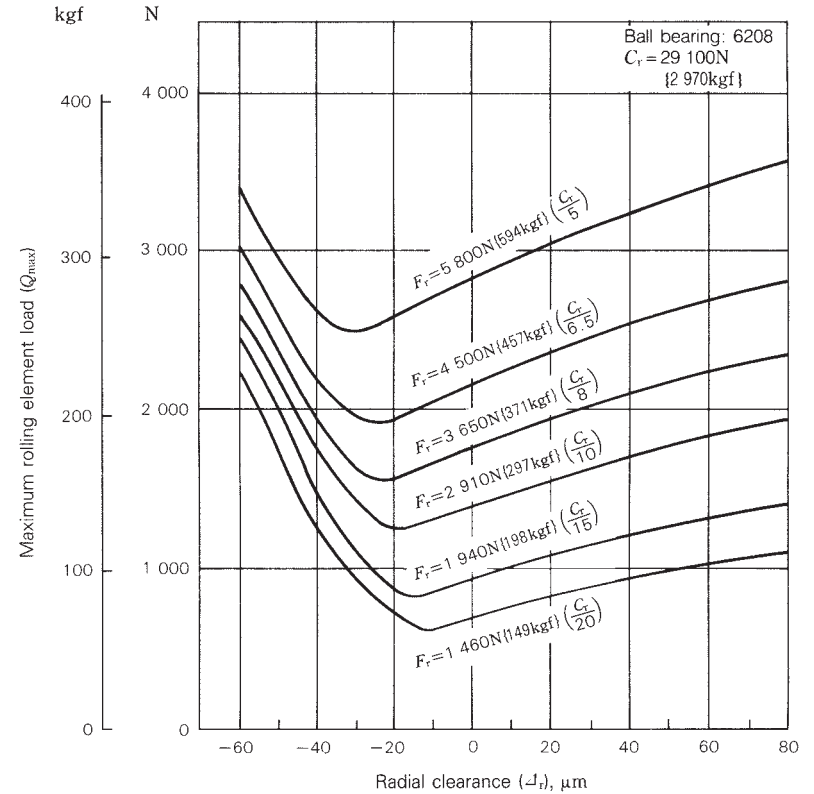


Fig. 2 Radial clearance and maximum rolling element load

**5.4 Contact surface pressure and contact ellipse of ball bearings under pure radial loads**

Details about the contact between a rolling element and raceway is a classic exercise in the Hertz theory and one where theory and practice have proven to agree well. It also forms the basis for theories on ball bearing life and friction.

Generally, the contact conditions between the inner ring raceway and ball is more severe than those between the outer ring raceway and ball. Moreover, when checking the running trace (rolling contact trace), it is much easier to observe the inner ring raceway than the outer ring raceway. Therefore, we explain the relation of contact ellipse width and load between an inner ring raceway and a ball in a deep groove ball bearing. With no applied load, the ball and inner ring raceway meet at a point. When a load is applied to the bearing, however, elastic deformation is caused and the contact area assumes an elliptical shape as shown in Fig. 1.

When a ball bearing is subjected to a load, the resulting maximum contact surface pressure on the elliptical area of contact between a ball and a bearing raceway is  $P_{max}$ . The major axis of the ellipse is represented by  $2a$  and the minor axis by  $2b$ . The following relationships were derived from the Hertz equation.

$$P_{max} = \frac{1.5}{\pi} \left\{ \frac{3}{E} \left( 1 - \frac{1}{m^2} \right) \right\}^{-2/3} \frac{1}{\mu\nu} (\Sigma\rho)^{2/3} Q^{1/3}$$

$$= \frac{A_1}{\mu\nu} (\Sigma\rho)^{2/3} Q^{1/3}$$

(MPa), {kgf/mm<sup>2</sup>} ..... (1)

where, Constant  $A_1$ : 858 for (N-unit), 187 for {kgf-unit}

$$2a = \mu \left\{ \frac{24 \left( 1 - \frac{1}{m^2} \right) Q}{E\Sigma\rho} \right\}^{1/3}$$

$$= A_2 \mu \left( \frac{Q}{\Sigma\rho} \right)^{1/3} \quad (\text{mm}) \dots\dots\dots (2)$$

where, Constant  $A_2$ : 0.0472 for (N-unit), 0.101 for {kgf-unit}

$$2b = \nu \left\{ \frac{24 \left( 1 - \frac{1}{m^2} \right) Q}{E\Sigma\rho} \right\}^{1/3}$$

$$= A_2 \nu \left( \frac{Q}{\Sigma\rho} \right)^{1/3} \quad (\text{mm}) \dots\dots\dots (3)$$

where,  $E$ : Young's modulus (Steel:  $E=208\,000$  MPa {21 200 kgf/mm<sup>2</sup>})  
 $m$ : Poisson's number (Steel: 10/3)  
 $Q$ : Rolling element (ball) load (N), {kgf}  
 $\Sigma\rho$ : Total major curvature

For radial ball bearing,

$$\Sigma\rho = \frac{1}{D_w} \left( 4 - \frac{1}{f} \pm \frac{2\gamma}{1 \mp \gamma} \right) \dots\dots\dots (4)$$

Symbol of  $\pm$ : The upper is for inner ring.  
 The lower is for outer ring.

- $D_w$ : Ball diameter (mm)
- $f$ : Ratio of groove radius to ball diameter
- $\gamma$ :  $D_w \cos\alpha / D_{pw}$
- $D_{pw}$ : Ball pitch diameter (mm)
- $\alpha$ : Contact angle (°)

$\mu$  and  $\nu$  are shown in Fig. 2 based on  $\cos\tau$  in Equation (5).

$$\cos\tau = \frac{1 \pm \frac{2\gamma}{1 \mp \gamma}}{4 - \frac{1}{f} \pm \frac{2\gamma}{1 \mp \gamma}} \dots\dots\dots (5)$$

Symbol of  $\pm$ : The upper is for the inner ring.  
 The lower is for the outer ring.

If the maximum rolling element load of the ball bearing under the radial load  $F_r$  is  $Q_{max}$  and the number of balls is  $Z$ , an approximate relation between them is shown in Equation (6).

$$Q_{max} = 5 \frac{F_r}{Z} \dots\dots\dots (6)$$

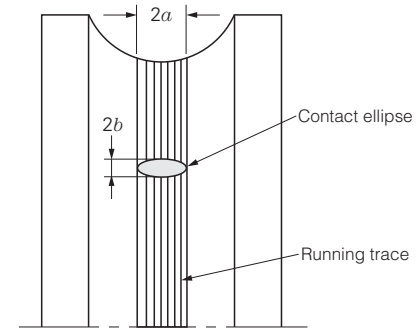


Fig. 1 Inner ring raceway running trace (Rolling contact trace)

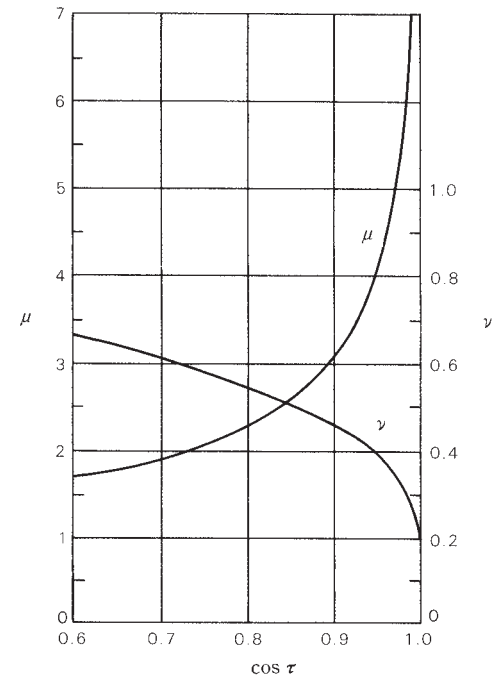


Fig. 2  $\mu$  and  $\nu$  values against  $\cos\tau$

Therefore, Equations (1), (2), and (3) can be changed into the following equations by substituting Equations (4) and (6).

$$\left. \begin{aligned} P_{\max} &= K_1 \cdot F_r^{1/3} && \text{(MPa)} \\ &= 0.218 K_1 \cdot F_r^{1/3} && \text{(kgf/mm}^2\text{)} \end{aligned} \right\} \dots\dots\dots (7)$$

$$\left. \begin{aligned} 2a &= K_2 \cdot F_r^{1/3} && \text{(N)} \\ &= 2.14 K_2 \cdot F_r^{1/3} && \text{(kgf)} \end{aligned} \right\} \text{(mm)} \dots\dots\dots (8)$$

$$\left. \begin{aligned} 2b &= K_3 \cdot F_r^{1/3} && \text{(N)} \\ &= 2.14 K_3 \cdot F_r^{1/3} && \text{(kgf)} \end{aligned} \right\} \text{(mm)} \dots\dots\dots (9)$$

**Table 1** gives values for the constants  $K_1$ ,  $K_2$ , and  $K_3$  for different bearing numbers.

Generally, the ball bearing raceway has a running trace caused by the balls whose width is equivalent to  $2a$ . We can estimate the applied load by referring to the trace on the raceway. Therefore, we can judge whether or not any abnormal load was sustained by the bearing which was beyond what the bearing was originally designed to carry.

**Example**

The pure radial load,  $F_r=3\ 500$  N (10% of basic dynamic load rating), is applied to a deep groove ball bearing, 6210. Calculate the maximum surface pressure,  $P_{\max}$ , and contact widths of the ball and inner ring raceway,  $2a$  and  $2b$ .

Using the figures of  $K_1 \sim K_3$  in **Table 1**, the following values can be obtained.

$$\begin{aligned} P_{\max} &= K_1 \cdot F_r^{1/3} = 143 \times 3\ 500^{1/3} = 2\ 170 \text{ (MPa)} \\ 2a &= K_2 \cdot F_r^{1/3} = 0.258 \times 3\ 500^{1/3} = 3.92 \text{ (mm)} \\ 2b &= K_3 \cdot F_r^{1/3} = 0.026 \times 3\ 500^{1/3} = 0.39 \text{ (mm)} \end{aligned}$$

**Table 1** Values of constants,

| Bearing bore No. | Bearing series 60 |       |       |
|------------------|-------------------|-------|-------|
|                  | $K_1$             | $K_2$ | $K_3$ |
| 00               | 324               | 0.215 | 0.020 |
| 01               | 305               | 0.205 | 0.019 |
| 02               | 287               | 0.196 | 0.019 |
| 03               | 274               | 0.189 | 0.018 |
| 04               | 191               | 0.332 | 0.017 |
| 05               | 181               | 0.320 | 0.016 |
| 06               | 160               | 0.326 | 0.017 |
| 07               | 148               | 0.342 | 0.017 |
| 08               | 182               | 0.205 | 0.021 |
| 09               | 166               | 0.206 | 0.021 |
| 10               | 161               | 0.201 | 0.021 |
| 11               | 148               | 0.219 | 0.023 |
| 12               | 144               | 0.214 | 0.022 |
| 13               | 140               | 0.209 | 0.022 |
| 14               | 130               | 0.224 | 0.023 |
| 15               | 127               | 0.219 | 0.023 |
| 16               | 120               | 0.235 | 0.024 |
| 17               | 117               | 0.229 | 0.024 |
| 18               | 111               | 0.244 | 0.025 |
| 19               | 108               | 0.238 | 0.025 |
| 20               | 108               | 0.238 | 0.025 |
| 21               | 102               | 0.243 | 0.026 |
| 22               | 98.2              | 0.268 | 0.028 |
| 24               | 95.3              | 0.261 | 0.027 |
| 26               | 88.1              | 0.263 | 0.028 |
| 28               | 85.9              | 0.257 | 0.027 |
| 30               | 81.8              | 0.264 | 0.028 |

$K_1$ ,  $K_2$ , and  $K_3$ , for deep groove ball bearings

|      | Bearing series 62 |       |       | Bearing series 63 |       |       |
|------|-------------------|-------|-------|-------------------|-------|-------|
|      | $K_1$             | $K_2$ | $K_3$ | $K_1$             | $K_2$ | $K_3$ |
| 303  | 0.205             | 0.019 | 0.019 | 215               | 0.404 | 0.018 |
| 226  | 0.352             | 0.017 | 0.017 | 200               | 0.423 | 0.019 |
| 211  | 0.336             | 0.017 | 0.017 | 184               | 0.401 | 0.019 |
| 193  | 0.356             | 0.017 | 0.017 | 171               | 0.415 | 0.019 |
| 172  | 0.382             | 0.018 | 0.018 | 161               | 0.431 | 0.020 |
| 162  | 0.367             | 0.018 | 0.018 | 142               | 0.426 | 0.020 |
| 143  | 0.395             | 0.019 | 0.019 | 129               | 0.450 | 0.021 |
| 128  | 0.420             | 0.020 | 0.020 | 118               | 0.474 | 0.021 |
| 157  | 0.262             | 0.026 | 0.026 | 112               | 0.469 | 0.023 |
| 150  | 0.252             | 0.025 | 0.025 | 129               | 0.308 | 0.030 |
| 143  | 0.258             | 0.026 | 0.026 | 122               | 0.318 | 0.031 |
| 133  | 0.269             | 0.027 | 0.027 | 116               | 0.327 | 0.032 |
| 124  | 0.275             | 0.028 | 0.028 | 110               | 0.336 | 0.032 |
| 120  | 0.280             | 0.028 | 0.028 | 105               | 0.344 | 0.033 |
| 116  | 0.284             | 0.029 | 0.029 | 100               | 0.352 | 0.034 |
| 112  | 0.275             | 0.028 | 0.028 | 96.5              | 0.356 | 0.035 |
| 109  | 0.293             | 0.030 | 0.030 | 92.8              | 0.364 | 0.035 |
| 104  | 0.302             | 0.031 | 0.031 | 89.4              | 0.371 | 0.036 |
| 98.7 | 0.310             | 0.031 | 0.031 | 86.3              | 0.377 | 0.037 |
| 94.3 | 0.318             | 0.032 | 0.032 | 83.4              | 0.384 | 0.037 |
| 90.3 | 0.325             | 0.033 | 0.033 | 78.6              | 0.394 | 0.038 |
| 87.2 | 0.329             | 0.033 | 0.033 | 76.7              | 0.400 | 0.039 |
| 83.9 | 0.336             | 0.034 | 0.034 | 72.7              | 0.412 | 0.040 |
| 80.7 | 0.343             | 0.035 | 0.035 | 72.0              | 0.411 | 0.040 |
| 77.8 | 0.349             | 0.035 | 0.035 | 68.5              | 0.422 | 0.041 |
| 77.2 | 0.348             | 0.036 | 0.036 | 65.5              | 0.431 | 0.042 |
| 74.3 | 0.337             | 0.035 | 0.035 | 62.5              | 0.414 | 0.041 |

**5.5 Contact surface pressure and contact area under pure radial load (roller bearings)**

The following equations, Equations 1 and 2, which were derived from the Hertz equation, give the contact surface pressure  $P_{max}$  between two axially-parallel cylinders and the contact area width  $2b$  (Fig. 1).

$$P_{max} = \sqrt{\frac{E \cdot \Sigma\rho \cdot Q}{2\pi \left(1 - \frac{1}{m^2}\right) L_{we}}} = A_1 \sqrt{\frac{\Sigma\rho \cdot Q}{L_{we}}} \quad \text{(MPa) \{kgf/mm}^2\}} \quad \dots\dots\dots (1)$$

where, constant  $A_1$ : 191 ..... (N-unit)  
: 60.9 ..... (kgf-unit)

$$2b = \sqrt{\frac{32 \left(1 - \frac{1}{m^2}\right) Q}{\pi \cdot E \cdot \Sigma\rho \cdot L_{we}}} = A_2 \sqrt{\frac{Q}{\Sigma\rho \cdot L_{we}}} \quad \text{(mm)} \quad \dots\dots\dots (2)$$

where, constant  $A_2$ : 0.00668 ..... (N-unit)  
: 0.0209 ..... (kgf-unit)

- where,  $E$ : Young's modulus (Steel:  $E=208\ 000$  MPa {21 200 kgf/mm<sup>2</sup>})
- $m$ : Poisson's number (for steel,  $m=10/3$ )
- $\Sigma\rho$ : Composite curvature for both cylinders  
 $\Sigma\rho = \rho_{II} + \rho_{III}$  (mm<sup>-1</sup>)
- $\rho_{II}$ : Curvature, cylinder I (roller)  
 $\rho_{II} = 1/D_w/2 = 2/D_w$  (mm<sup>-1</sup>)
- $\rho_{III}$ : Curvature, cylinder II (raceway)  
 $\rho_{III} = 1/D_i/2 = 2/D_i$  (mm<sup>-1</sup>) for inner ring raceway  
 $\rho_{III} = -1/D_o/2 = -2/D_o$  (mm<sup>-1</sup>) for outer ring raceway
- $Q$ : Normal load on cylinders (N), {kgf}
- $L_{we}$ : Effective contact length of cylinders (mm)

When a radial load  $F_r$  is applied on a radial roller bearing, the maximum rolling element load  $Q_{max}$  for practical use is given by Equation (3).

$$Q_{max} = \frac{4.6 F_r}{i Z \cos\alpha} \quad \text{(N), \{kgf\}} \quad \dots\dots\dots (3)$$

where,  $i$ : Number of roller rows  
 $Z$ : Number of rollers per row  
 $\alpha$ : Contact angle (°)

The contact surface pressure  $P_{max}$  and contact width  $2b$  of raceway and roller which sustains the largest load are given by Equations (4) and (5).

$$\left. \begin{aligned} P_{max} &= K_1 \sqrt{F_r} && \text{(MPa)} \\ &= 0.319K_1 \sqrt{F_r} && \text{\{kgf/mm}^2\}} \end{aligned} \right\} \quad \dots\dots\dots (4)$$

$$\left. \begin{aligned} 2b &= K_2 \sqrt{F_r} && \text{(N)} \\ &= 3.13K_2 \sqrt{F_r} && \text{\{kgf\}} \end{aligned} \right\} \quad \dots\dots\dots (5)$$

The constant  $K_1$  and  $K_2$  of cylindrical roller bearings and tapered roller bearings are listed in Tables 1 to 6 according to the bearing numbers.  $K_{1i}$  and  $K_{2i}$  are the constants for the contact of roller and inner ring raceway, and  $K_{1e}$  and  $K_{2e}$  are the constants for the contact of roller and outer ring raceway.

**Example**

A pure radial load,  $F_r=4\ 800$  N (10% of basic dynamic load rating), is applied to the cylindrical roller bearing, NU210. Calculate the maximum surface pressure,  $P_{max}$ , and contact widths of the roller and raceway,  $2b$ .

Using the figures of  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$  in Table 1, the following values can be obtained.

Contact of roller and inner ring raceway:  
 $P_{max} = K_{1i} \sqrt{F_r} = 17.0 \times \sqrt{4\ 800} = 1\ 180$  (MPa)  
 $2b = K_{2i} \sqrt{F_r} = 2.55 \times 10^{-3} \times \sqrt{4\ 800} = 0.18$  (mm)

Contact of roller and outer ring raceway:  
 $P_{max} = K_{1e} \sqrt{F_r} = 14.7 \times \sqrt{4\ 800} = 1\ 020$  (MPa)  
 $2b = K_{2e} \sqrt{F_r} = 2.95 \times 10^{-3} \times \sqrt{4\ 800} = 0.20$  (mm)

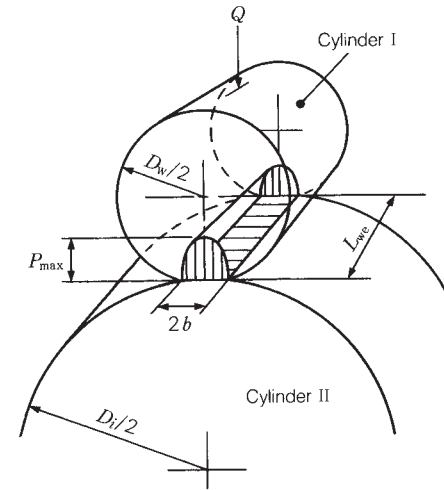


Fig. 1 Contact surface pressure  $P_{max}$  and contact width  $2b$

Table 1 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for cylindrical roller bearings

| Bearing No. | Bearing series NU2 |          |                  |                  | Bearing No. | Bearing series NU3 |          |                  |                  |
|-------------|--------------------|----------|------------------|------------------|-------------|--------------------|----------|------------------|------------------|
|             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| NU205W      | 30.6               | 25.8     | 2.90             | 3.44             | NU305W      | 24.2               | 19.6     | 3.03             | 3.73             |
| NU206W      | 26.1               | 22.2     | 2.87             | 3.39             | NU306W      | 20.5               | 16.8     | 2.89             | 3.52             |
| NU207W      | 21.6               | 18.2     | 2.83             | 3.36             | NU307W      | 17.7               | 14.6     | 2.76             | 3.35             |
| NU208W      | 18.5               | 15.7     | 2.70             | 3.20             | NU308W      | 16.1               | 13.4     | 2.76             | 3.32             |
| NU209W      | 17.7               | 15.2     | 2.63             | 3.07             | NU309W      | 14.4               | 11.8     | 2.85             | 3.46             |
| NU210W      | 17.0               | 14.7     | 2.55             | 2.95             | NU310W      | 13.1               | 10.8     | 2.79             | 3.37             |
| NU211W      | 15.4               | 13.3     | 2.54             | 2.93             | NU311W      | 11.5               | 9.44     | 2.76             | 3.36             |
| NU212W      | 14.0               | 12.2     | 2.53             | 2.92             | NU312W      | 10.8               | 8.91     | 2.76             | 3.34             |
| NU213W      | 12.5               | 10.8     | 2.44             | 2.82             | NU313W      | 10.3               | 8.54     | 2.79             | 3.37             |
| NU214W      | 12.4               | 10.9     | 2.45             | 2.81             | NU314W      | 9.35               | 7.78     | 2.68             | 3.22             |
| NU215W      | 11.5               | 10.1     | 2.44             | 2.80             | NU315W      | 8.83               | 7.31     | 2.77             | 3.34             |
| NU216W      | 11.0               | 9.57     | 2.49             | 2.86             | NU316W      | 8.43               | 7.05     | 2.68             | 3.20             |
| NU217W      | 10.2               | 8.94     | 2.48             | 2.85             | NU317W      | 8.04               | 6.68     | 2.76             | 3.32             |
| NU218W      | 9.10               | 7.87     | 2.45             | 2.84             | NU318W      | 7.45               | 6.22     | 2.68             | 3.21             |
| NU219W      | 8.98               | 7.77     | 2.56             | 2.96             | NU319W      | 7.14               | 5.97     | 2.68             | 3.20             |
| NU220W      | 8.23               | 7.13     | 2.47             | 2.85             | NU320W      | 6.61               | 5.52     | 2.66             | 3.19             |
| NU221W      | 7.82               | 6.78     | 2.47             | 2.85             | NU321W      | 6.42               | 5.34     | 2.76             | 3.31             |
| NU222W      | 7.36               | 6.34     | 2.53             | 2.93             | NU322W      | 6.06               | 5.04     | 2.78             | 3.34             |
| NU224W      | 7.02               | 6.08     | 2.53             | 2.92             | NU324W      | 5.38               | 4.44     | 2.75             | 3.33             |
| NU226W      | 6.76               | 5.91     | 2.46             | 2.82             | NU326W      | 5.07               | 4.21     | 2.75             | 3.32             |
| NU228W      | 6.27               | 5.48     | 2.47             | 2.83             | NU328W      | 4.80               | 3.99     | 2.75             | 3.31             |
| NU230W      | 5.80               | 5.07     | 2.47             | 2.83             | NU330W      | 4.61               | 3.85     | 2.79             | 3.34             |

Table 2 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for cylindrical roller bearings

| Bearing No. | Bearing series NU4 |          |                  |                  | Bearing No. | Bearing series NU22 |          |                  |                  |
|-------------|--------------------|----------|------------------|------------------|-------------|---------------------|----------|------------------|------------------|
|             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$            | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                     |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| NU405W      | 19.2               | 15.1     | 3.08             | 3.92             | NU2205W     | 25.4                | 21.4     | 2.40             | 2.85             |
| NU406W      | 16.4               | 12.9     | 3.06             | 3.90             | NU2206W     | 21.1                | 17.9     | 2.32             | 2.73             |
| NU407W      | 14.6               | 11.7     | 2.99             | 3.74             | NU2207W     | 17.0                | 14.3     | 2.22             | 2.63             |
| NU408W      | 12.9               | 10.2     | 2.96             | 3.73             | NU2208W     | 15.4                | 13.0     | 2.25             | 2.66             |
| NU409W      | 12.0               | 9.65     | 2.97             | 3.70             | NU2209W     | 14.7                | 12.6     | 2.18             | 2.55             |
| NU410W      | 10.9               | 8.73     | 2.98             | 3.73             | NU2210W     | 14.1                | 12.3     | 2.12             | 2.45             |
| NU411W      | 10.3               | 8.37     | 2.87             | 3.54             | NU2211W     | 13.0                | 11.3     | 2.15             | 2.48             |
| NU412W      | 9.35               | 7.56     | 2.85             | 3.52             | NU2212W     | 11.3                | 9.79     | 2.04             | 2.35             |
| NU413W      | 8.90               | 7.23     | 2.85             | 3.51             | NU2213W     | 9.93                | 8.62     | 1.94             | 2.24             |
| NU414W      | 7.90               | 6.41     | 2.86             | 3.52             | NU2214W     | 9.88                | 8.64     | 1.95             | 2.23             |
| NU415W      | 7.34               | 5.92     | 2.84             | 3.52             | NU2215W     | 9.54                | 8.32     | 2.02             | 2.32             |
| NU416W      | 6.84               | 5.50     | 2.82             | 3.51             | NU2216W     | 8.90                | 7.76     | 2.02             | 2.31             |
| NU417M      | 6.49               | 5.18     | 2.83             | 3.55             | NU2217W     | 8.22                | 7.17     | 1.99             | 2.28             |
| NU418M      | 6.07               | 4.87     | 2.83             | 3.53             | NU2218W     | 7.46                | 6.45     | 2.01             | 2.33             |
| NU419M      | 5.76               | 4.69     | 2.73             | 3.36             | NU2219W     | 7.03                | 6.08     | 2.00             | 2.32             |
| NU420M      | 5.44               | 4.41     | 2.72             | 3.35             | NU2220W     | 6.82                | 5.90     | 2.05             | 2.36             |
| NU421M      | 5.15               | 4.17     | 2.71             | 3.35             | NU2221M     | 6.44                | 5.58     | 2.03             | 2.34             |
| NU422M      | 4.87               | 3.95     | 2.71             | 3.34             | NU2222W     | 5.96                | 5.14     | 2.05             | 2.38             |
| NU424M      | 4.37               | 3.54     | 2.72             | 3.37             | NU2224W     | 5.65                | 4.89     | 2.03             | 2.35             |
| NU426M      | 3.92               | 3.16     | 2.71             | 3.36             | NU2226W     | 5.28                | 4.61     | 1.92             | 2.20             |
| NU428M      | 3.80               | 3.07     | 2.74             | 3.38             | NU2228W     | 4.82                | 4.22     | 1.90             | 2.18             |
| NU430M      | 2.97               | 2.97     | 2.65             | 3.23             | NU2230W     | 4.55                | 3.98     | 1.93             | 2.21             |

Table 3 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for cylindrical roller bearings

| Bearing No. | Bearing series NU23 |          |                  |                  | Bearing No. | Bearing series NN30 |          |                  |                  |
|-------------|---------------------|----------|------------------|------------------|-------------|---------------------|----------|------------------|------------------|
|             | $K_{1i}$            | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$            | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                     |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                     |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| NU2305W     | 19.0                | 15.4     | 2.38             | 2.93             | NN3005      | 31.3                | 27.3     | 2.36             | 2.72             |
| NU2306W     | 17.0                | 14.0     | 2.41             | 2.93             | NN3006      | 28.1                | 24.7     | 2.36             | 2.69             |
| NU2307W     | 15.6                | 12.9     | 2.43             | 2.96             | NN3007T     | 24.3                | 21.5     | 2.24             | 2.53             |
| NU2308W     | 12.9                | 10.7     | 2.22             | 2.67             | NN3008T     | 23.1                | 20.4     | 2.31             | 2.61             |
| NU2309W     | 11.9                | 9.79     | 2.36             | 2.86             | NN3009T     | 20.7                | 18.4     | 2.25             | 2.52             |
| NU2310W     | 10.6                | 8.76     | 2.26             | 2.73             | NN3010T     | 20.1                | 18.1     | 2.20             | 2.45             |
| NU2311W     | 9.53                | 7.83     | 2.29             | 2.78             | NN3011T     | 17.5                | 15.6     | 2.18             | 2.43             |
| NU2312W     | 8.85                | 7.31     | 2.26             | 2.74             | NN3012T     | 16.7                | 15.0     | 2.09             | 2.32             |
| NU2313W     | 8.32                | 6.90     | 2.26             | 2.72             | NN3013T     | 15.9                | 14.5     | 2.02             | 2.22             |
| NU2314W     | 7.50                | 6.24     | 2.15             | 2.58             | NN3014T     | 14.4                | 13.0     | 2.04             | 2.25             |
| NU2315W     | 6.98                | 5.78     | 2.19             | 2.64             | NN3015T     | 14.0                | 12.8     | 2.01             | 2.20             |
| NU2316W     | 6.66                | 5.58     | 2.11             | 2.53             | NN3016T     | 12.6                | 11.4     | 1.99             | 2.19             |
| NU2317W     | 6.21                | 5.17     | 2.14             | 2.57             | NN3017T     | 12.3                | 11.2     | 1.96             | 2.15             |
| NU2318W     | 6.11                | 5.10     | 2.20             | 2.63             | NN3018T     | 11.4                | 10.3     | 1.98             | 2.18             |
| NU2319W     | 5.65                | 4.73     | 2.12             | 2.53             | NN3019T     | 11.1                | 10.2     | 1.95             | 2.14             |
| NU2320W     | 5.40                | 4.51     | 2.18             | 2.60             | NN3020T     | 10.9                | 10.0     | 1.92             | 2.09             |
| NU2321M     | 4.80                | 3.99     | 2.06             | 2.48             | NN3021T     | 9.75                | 8.84     | 2.00             | 2.21             |
| NU2322M     | 4.48                | 3.73     | 2.05             | 2.47             | NN3022T     | 9.04                | 8.18     | 2.00             | 2.20             |
| NU2324M     | 4.00                | 3.31     | 2.05             | 2.48             | NN3024T     | 8.66                | 7.90     | 1.93             | 2.11             |
| NU2326M     | 3.62                | 3.00     | 1.96             | 2.37             | NN3026T     | 7.86                | 7.14     | 1.99             | 2.19             |
| NU2328M     | 3.43                | 2.86     | 1.97             | 2.36             | NN3028      | 7.55                | 6.90     | 1.92             | 2.11             |
| NU2330M     | 3.24                | 2.70     | 1.96             | 2.34             | NN3030      | 7.08                | 6.47     | 1.92             | 2.10             |

Table 4 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for tapered roller bearings

| Bearing No. | Bearing series 302 |          |                  |                  | Bearing No. | Bearing series 303 |          |                  |                  |
|-------------|--------------------|----------|------------------|------------------|-------------|--------------------|----------|------------------|------------------|
|             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| HR30205J    | 20.6               | 17.4     | 1.94             | 2.29             | HR30305J    | 17.8               | 14.3     | 2.34             | 2.92             |
| HR30206J    | 17.7               | 14.9     | 1.99             | 2.36             | HR30306J    | 15.7               | 12.8     | 2.30             | 2.83             |
| HR30207J    | 15.8               | 13.3     | 2.07             | 2.45             | HR30307J    | 13.7               | 11.1     | 2.26             | 2.78             |
| HR30208J    | 14.5               | 12.3     | 2.13             | 2.52             | HR30308J    | 12.1               | 10.0     | 2.09             | 2.51             |
| HR30209J    | 13.7               | 11.7     | 2.03             | 2.37             | HR30309J    | 10.9               | 9.07     | 2.11             | 2.54             |
| HR30210J    | 12.7               | 11.0     | 1.96             | 2.28             | HR30310J    | 10.1               | 8.37     | 2.16             | 2.60             |
| HR30211J    | 11.4               | 9.80     | 2.02             | 2.36             | HR30311J    | 9.38               | 7.79     | 2.19             | 2.64             |
| HR30212J    | 11.0               | 9.41     | 2.11             | 2.46             | HR30312J    | 8.66               | 7.19     | 2.19             | 2.64             |
| HR30213J    | 10.0               | 8.62     | 2.05             | 2.38             | HR30313J    | 8.04               | 6.68     | 2.20             | 2.65             |
| HR30214J    | 9.62               | 8.28     | 2.07             | 2.40             | HR30314J    | 7.49               | 6.22     | 2.20             | 2.65             |
| HR30215J    | 9.11               | 7.89     | 1.99             | 2.30             | HR30315J    | 7.09               | 5.88     | 2.23             | 2.68             |
| HR30216J    | 8.79               | 7.57     | 2.12             | 2.47             | HR30316J    | 6.79               | 5.64     | 2.28             | 2.74             |
| HR30217J    | 8.04               | 6.93     | 2.07             | 2.40             | HR30317J    | 6.30               | 5.24     | 2.22             | 2.68             |
| HR30218J    | 7.69               | 6.63     | 2.10             | 2.44             | 30318       | 6.42               | 5.34     | 2.41             | 2.89             |
| HR30219J    | 7.27               | 6.26     | 2.11             | 2.45             | 30319       | 6.09               | 5.06     | 2.37             | 2.85             |
| HR30220J    | 6.74               | 5.81     | 2.07             | 2.40             | 30320       | 5.84               | 4.86     | 2.43             | 2.92             |
| HR30221J    | 6.36               | 5.48     | 2.06             | 2.39             | 30321       | 5.62               | 4.67     | 2.44             | 2.94             |
| HR30222J    | 5.94               | 5.12     | 2.03             | 2.36             | HR30322J    | 4.99               | 4.15     | 2.33             | 2.81             |
| HR30224J    | 5.74               | 4.97     | 2.06             | 2.38             | HR30324J    | 4.75               | 3.95     | 2.39             | 2.88             |
| 30226       | 5.83               | 5.07     | 2.23             | 2.57             | 30326       | 4.69               | 3.93     | 2.46             | 2.94             |
| HR30228J    | 5.36               | 4.64     | 2.24             | 2.58             | 30328       | 4.47               | 3.75     | 2.50             | 2.98             |
| 30230       | 5.10               | 4.41     | 2.31             | 2.67             | 30330       | 4.15               | 3.48     | 2.50             | 2.98             |



Table 5 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for tapered roller bearings

| Bearing No. | Bearing series 322 |          |                  |                  | Bearing No. | Bearing series 323 |          |                  |                  |
|-------------|--------------------|----------|------------------|------------------|-------------|--------------------|----------|------------------|------------------|
|             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| HR32205     | 18.5               | 15.6     | 1.72             | 2.04             | HR32305J    | 15.0               | 12.0     | 1.93             | 2.40             |
| HR32206J    | 15.7               | 13.2     | 1.76             | 2.08             | HR32306J    | 12.9               | 10.5     | 1.86             | 2.28             |
| HR32207J    | 13.3               | 11.2     | 1.73             | 2.05             | HR32307J    | 11.5               | 9.38     | 1.87             | 2.30             |
| HR32208J    | 12.8               | 10.8     | 1.88             | 2.22             | HR32308J    | 10.1               | 8.38     | 1.71             | 2.06             |
| HR32209J    | 12.0               | 10.3     | 1.79             | 2.09             | HR32309J    | 9.22               | 7.65     | 1.75             | 2.11             |
| HR32210J    | 11.7               | 10.0     | 1.80             | 2.08             | HR32310J    | 8.26               | 6.86     | 1.73             | 2.08             |
| HR32211J    | 10.4               | 8.90     | 1.83             | 2.14             | HR32311J    | 7.62               | 6.33     | 1.74             | 2.10             |
| HR32212J    | 9.43               | 8.08     | 1.80             | 2.10             | HR32312J    | 7.13               | 5.92     | 1.77             | 2.13             |
| HR32213J    | 9.64               | 7.40     | 1.82             | 2.13             | HR32313J    | 6.62               | 5.50     | 1.78             | 2.15             |
| HR32214J    | 8.58               | 7.39     | 1.84             | 2.14             | HR32314J    | 6.21               | 5.16     | 1.79             | 2.16             |
| HR32215J    | 8.28               | 7.18     | 1.81             | 2.09             | HR32315J    | 5.80               | 4.81     | 1.79             | 2.15             |
| HR32216J    | 7.70               | 6.63     | 1.86             | 2.15             | HR32316J    | 5.46               | 4.54     | 1.80             | 2.16             |
| HR32217J    | 7.38               | 6.36     | 1.90             | 2.21             | HR32317J    | 5.26               | 4.36     | 1.83             | 2.20             |
| HR32218J    | 6.56               | 5.65     | 1.80             | 2.09             | HR32318J    | 5.00               | 4.15     | 1.83             | 2.20             |
| HR32219J    | 6.14               | 5.29     | 1.78             | 2.07             | 32319       | 4.97               | 4.13     | 1.89             | 2.27             |
| HR32220J    | 5.77               | 4.97     | 1.77             | 2.06             | HR32320J    | 4.43               | 3.68     | 1.84             | 2.21             |
| HR32221J    | 5.39               | 4.64     | 1.74             | 2.02             | 32321       | 4.36               | 3.62     | 1.88             | 2.27             |
| HR32222J    | 5.12               | 4.41     | 1.75             | 2.03             | HR32322J    | 4.03               | 3.35     | 1.87             | 2.25             |
| HR32224J    | 4.82               | 4.18     | 1.72             | 1.98             | HR32324J    | 3.75               | 3.11     | 1.87             | 2.25             |
| 32226       | 4.48               | 3.90     | 1.68             | 1.93             | 32326       | 3.59               | 3.01     | 1.89             | 2.26             |
| HR32228J    | 4.02               | 3.48     | 1.67             | 1.93             | 32328       | 3.21               | 2.71     | 1.75             | 2.08             |
| 32230       | 4.06               | 3.55     | 1.74             | 1.99             | 32330       | 2.95               | 2.51     | 1.65             | 1.94             |

Table 6 Constants,  $K_{1i}$ ,  $K_{1e}$ ,  $K_{2i}$ , and  $K_{2e}$ , for tapered roller bearings

| Bearing No. | Bearing series 303D |          |                  |                  | Bearing No. | Bearing series 320 |          |                  |                  |
|-------------|---------------------|----------|------------------|------------------|-------------|--------------------|----------|------------------|------------------|
|             | $K_{1i}$            | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |             | $K_{1i}$           | $K_{1e}$ | $K_{2i}$         | $K_{2e}$         |
|             |                     |          | $\times 10^{-3}$ | $\times 10^{-3}$ |             |                    |          | $\times 10^{-3}$ | $\times 10^{-3}$ |
| 30305D      | 22.0                | 18.4     | 2.42             | 2.91             | HR32005XJ   | 21.1               | 18.4     | 1.58             | 1.82             |
| 30306D      | 19.0                | 15.8     | 2.48             | 2.98             | HR32006XJ   | 18.2               | 15.9     | 1.61             | 1.85             |
| HR30307DJ   | 14.8                | 12.4     | 2.18             | 2.62             | HR32007XJ   | 16.4               | 14.4     | 1.57             | 1.79             |
| HR30308DJ   | 13.0                | 10.8     | 2.18             | 2.61             | HR32008XJ   | 14.4               | 12.7     | 1.48             | 1.67             |
| HR30309DJ   | 11.9                | 9.94     | 2.22             | 2.66             | HR32009XJ   | 13.3               | 11.8     | 1.47             | 1.65             |
| HR30310DJ   | 10.8                | 9.02     | 2.21             | 2.65             | HR32010XJ   | 13.0               | 11.6     | 1.45             | 1.62             |
| HR30311DJ   | 10.0                | 8.37     | 2.22             | 2.66             | HR32011XJ   | 11.3               | 10.0     | 1.46             | 1.64             |
| HR30312DJ   | 9.33                | 7.79     | 2.26             | 2.71             | HR32012XJ   | 10.8               | 9.69     | 1.41             | 1.57             |
| HR30313DJ   | 8.66                | 7.23     | 2.27             | 2.71             | HR32013XJ   | 10.6               | 9.57     | 1.39             | 1.54             |
| HR30314DJ   | 8.20                | 6.85     | 2.28             | 2.74             | HR32014XJ   | 9.68               | 8.70     | 1.44             | 1.60             |
| HR30315DJ   | 7.83                | 6.54     | 2.34             | 2.80             | HR32015XJ   | 9.32               | 8.43     | 1.39             | 1.54             |
| HR30316DJ   | 7.37                | 6.15     | 2.33             | 2.80             | HR32016XJ   | 8.15               | 7.35     | 1.36             | 1.51             |
| HR30317DJ   | 6.93                | 5.79     | 2.34             | 2.80             | HR32017XJ   | 8.00               | 7.25     | 1.34             | 1.48             |
| HR30318DJ   | 6.96                | 5.81     | 2.48             | 2.98             | HR32018XJ   | 7.36               | 6.64     | 1.37             | 1.52             |
| HR30319DJ   | 6.34                | 5.30     | 2.37             | 2.84             | HR32019XJ   | 7.22               | 6.54     | 1.35             | 1.50             |
| —           | —                   | —        | —                | —                | HR32020XJ   | 7.10               | 6.45     | 1.34             | 1.47             |
| —           | —                   | —        | —                | —                | HR32021XJ   | 6.61               | 5.99     | 1.36             | 1.50             |
| —           | —                   | —        | —                | —                | HR32022XJ   | 6.19               | 5.59     | 1.39             | 1.54             |
| —           | —                   | —        | —                | —                | HR32024XJ   | 6.10               | 5.52     | 1.42             | 1.56             |
| —           | —                   | —        | —                | —                | HR32026XJ   | 5.26               | 4.74     | 1.41             | 1.57             |
| —           | —                   | —        | —                | —                | HR32028XJ   | 5.15               | 4.67     | 1.39             | 1.54             |
| —           | —                   | —        | —                | —                | HR32030XJ   | 4.77               | 4.32     | 1.38             | 1.53             |

**5.6 Rolling contact trace and load conditions**

**5.6.1 Ball bearing**

When a rolling bearing is rotating while subjected to a load on the raceways of the inner and outer rings and on the surfaces of the rolling elements, heavy stress is generated at the place of contact. For example, when about 10% of the load (normal load) of the  $C_r$ , basic dynamic load rating, as radial load, is applied, in the case of deep groove ball bearings, its maximum surface pressure becomes about 2 000 MPa (204 kgf/mm<sup>2</sup>) and for a roller bearing, the pressure reaches 1 000 MPa (102 kgf/mm<sup>2</sup>).

As bearings are used under such high contact surface pressure, the contact parts of the rolling elements and raceway may become slightly elastically deformed or wearing may progress depending on lubrication conditions. As a result of this contact trace, light reflected from the raceway surface of a used bearing looks different for the places where a load was not applied.

Parts that were subjected to load reflect light differently and the dull appearance of such parts is called a trace (rolling contact trace). Thus, an examination of the trace can provide insights into the contact and load conditions.

Trace varies depending on the bearing type and conditions. Examination of the trace sometimes allows identification of the cause: radial load only, heavy axial load, moment load, or extreme unevenness of stiffness of housing.

For the case of a deep groove ball bearing used under an inner ring rotation load, only radial load  $F_r$  is applied, under the general condition of residual clearance after mounting is  $\Delta_r > 0$ , the load zone  $\psi$  becomes narrower than 180° (Fig. 1), traces on inner and outer rings become as shown in Fig. 2.

In addition to a radial load  $F_r$ , if an axial load  $F_a$  is simultaneously applied, the load zone  $\psi$  is widened as shown in Fig. 3. When only an axial load is applied, all rolling elements are uniformly subjected to the load, for both inner and outer rings the load zone becomes  $\psi=360^\circ$  and the race is unevenly displaced in the axial direction.

A deviated and inclined trace may be observed on the outer ring as shown in Fig. 4, when an axial load and relative inclination of the inner ring to the outer ring are applied together to a deep groove or angular contact ball bearing used for inner ring rotation load. Or if the deflection is big, a similar trace appears.

As explained above, by comparing the actual trace with the shape of the trace forecasted from the external force considered when the bearing was designed, it is possible to tell if an abnormal axial load was applied to the bearing or if the mounting error was excessive.

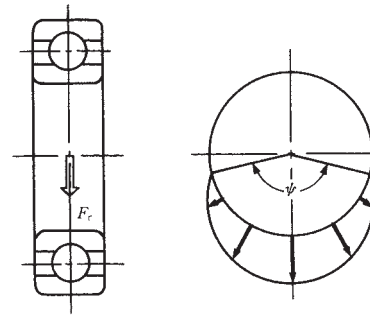


Fig. 1 Load zone under a radial load only

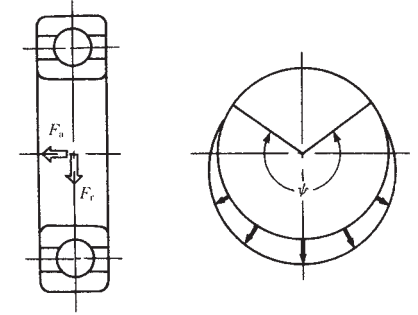


Fig. 3 Load zone for radial load + axial load

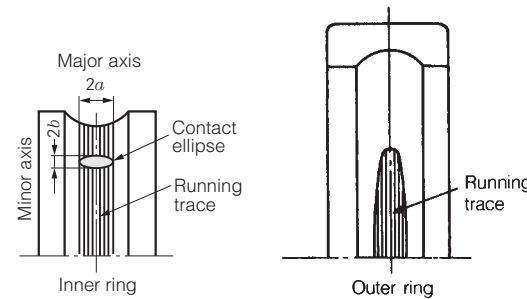


Fig. 2 Trace (rolling contact trace) on raceway surface

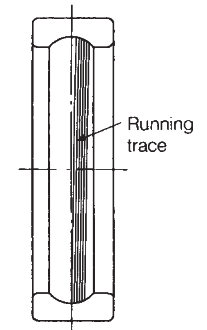


Fig. 4 Deviation and inclination of trace (rolling contact trace) on raceway surface of outer ring

**5.6.2 Roller bearing**

The relation between load condition and running trace of roller bearings may be described as follows. Usually, when rollers (or raceway) of a roller bearing are not crowned despite there being no relative inclination on inner ring with outer ring, then stress concentration occurs at the end parts where the rollers contact the raceway (Fig. 5 (a)). Noticeable contact appears at both ends of the trace. If the stress on the end parts is excessive, premature flaking occurs. Rollers (or raceway) can be crowned to reduce stress (Fig. 5 (b)). Even if the rollers are crowned, however, if inclination exists between the inner and outer rings, then stress at the contacting part becomes as shown in Fig. 5 (c).

Fig. 6 (a) shows an example of trace on an outer ring raceway for a radial load which is correctly applied to a cylindrical roller bearing and used for inner ring rotation. Compared with this, if there is relative inclination of inner ring to outer ring, as shown in Fig. 6 (b), the trace on raceway has shading in width direction. And the trace looks inclined at the entry and exit of the load zone.

The trace of outer ring becomes as shown in Fig. 7 (a) for double-row tapered roller bearings if only a radial load is applied while inner ring is rotating, or the trace becomes as shown in Fig. 7 (b) if only an axial load is applied. In addition, traces are produced on both sides of the raceway (displaced by 180°) as shown in Fig. 7 (c) if a radial load is applied under the condition that there is a large relative inclination of the inner ring to the outer ring.

The trace becomes even on the right and left sides of the raceways if a radial load is applied to a spherical roller bearing having the permissible aligning angle of 1 to 2.5°. In the case of an application of an axial load, the trace appears only on one side. A trace is produced that has a difference corresponding to them and is marked on right and left load zones if combined radial and axial loads are applied.

Therefore, the trace becomes even on the left and right sides for a free-end spherical roller bearing that is mainly subjected to radial load. If the length of the trace is greatly different, it indicates that internal axial load caused by thermal expansion of shaft, etc. was not sufficiently absorbed by displacement of the bearing in the axial direction.

Besides the above, the trace on raceway is influenced often by the shaft or housing. By comparison of the bearing outside face contact or pattern of fretting against the degree of trace on raceway, it is possible to tell if there is structural failure or uneven stiffness of shaft or housing.

As explained above, observation of the trace on raceway can help to prevent bearing trouble.

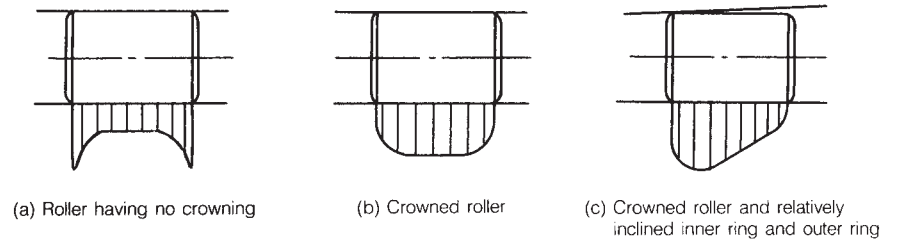


Fig. 5 Stress distribution of cylindrical roller

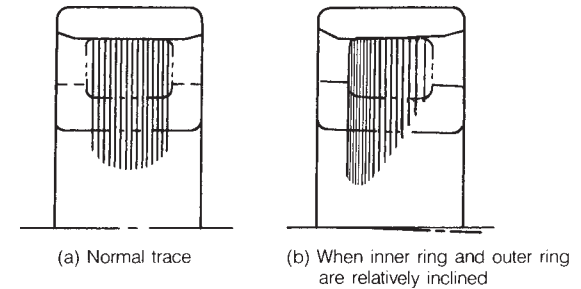


Fig. 6 Trace (rolling contact trace) of outer ring of cylindrical roller bearing

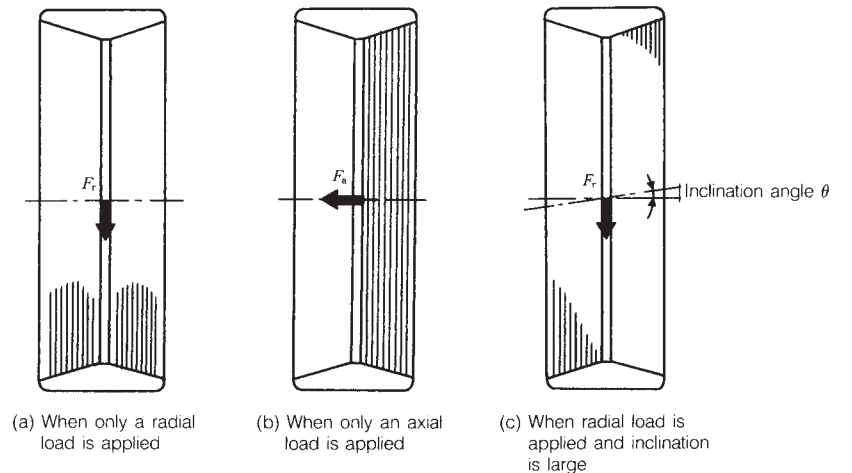


Fig. 7 Trace (rolling contact trace) of outer ring of double row tapered roller bearing

**5.7 Radial load and displacement of cylindrical roller bearings**

One of the most important requirements for bearings to be used in machine-tool applications is that there be as little deflection as possible with applied loading, i.e. that the bearings have high rigidity.

Double-row cylindrical roller bearings are considered to be the most rigid types under radial loads and also best for use at high speeds. NN30K and NNU49K series are the particular radial bearings most often used in machine tool head spindles.

The amount of bearing displacement under a radial load will vary with the amount of internal clearance in the bearings. However, since machine-tool spindle cylindrical roller bearings are adjusted so the internal clearance after mounting is less than several micrometers, we can consider the internal clearance to be zero for most general calculations. The radial elastic displacement  $\delta_r$  of cylindrical roller bearings can be calculated using Equation (1).

$$\left. \begin{aligned} \delta_r &= 0.000077 \frac{Q_{\max}^{0.9}}{L_{we}^{0.8}} \quad (\text{N}) \\ &= 0.0006 \frac{Q_{\max}^{0.9}}{L_{we}^{0.8}} \quad \{\text{kgf}\} \end{aligned} \right\} (\text{mm}) \dots\dots (1)$$

where,  $Q_{\max}$ : Maximum rolling element load (N), {kgf}  
 $L_{we}$ : Effective contact length of roller (mm)

If the internal clearance is zero, the relationship between maximum rolling element load  $Q_{\max}$  and radial load  $F_r$  becomes:

$$Q_{\max} = \frac{4.08}{iZ} F_r \quad (\text{N}), \{\text{kgf}\} \dots\dots\dots (2)$$

where,  $i$ : Number of rows of rollers in a bearing (double-row bearings:  $i=2$ )  
 $Z$ : Number of rollers per row  
 $F_r$ : Radial load (N), {kgf}

Combining Equations (1) and (2), it follows that the relation between radial load  $F_r$  and radial displacement  $\delta_r$  becomes.

$$\left. \begin{aligned} \delta_r &= K F_r^{0.9} \quad (\text{N}) \\ &= 7.8K F_r^{0.9} \quad \{\text{kgf}\} \end{aligned} \right\} (\text{mm}) \dots\dots (3)$$

where,  $K = \frac{0.000146}{Z^{0.9} L_{we}^{0.8}}$

$K$  is a constant determined by the individual double-row cylindrical roller bearing. Table 1 gives values for  $K$  for bearing series NN30. Fig. 1 shows the relation between radial load  $F_r$  and radial displacement  $\delta_r$ .

Table 1 Constant  $K$  for bearing series NN30

| Bearing | $K$              | Bearing | $K$              | Bearing | $K$              |
|---------|------------------|---------|------------------|---------|------------------|
|         | $\times 10^{-6}$ |         | $\times 10^{-6}$ |         | $\times 10^{-6}$ |
| NN3005  | 3.31             | NN3016T | 1.34             | NN3032  | 0.776            |
| NN3006T | 3.04             | NN3017T | 1.30             | NN3034  | 0.721            |
| NN3007T | 2.56             | NN3018T | 1.23             | NN3036  | 0.681            |
| NN3008T | 2.52             | NN3019T | 1.19             | NN3038  | 0.637            |
| NN3009T | 2.25             | NN3020T | 1.15             | NN3040  | 0.642            |
| NN3010T | 2.16             | NN3021T | 1.10             | NN3044  | 0.581            |
| NN3011T | 1.91             | NN3022T | 1.04             | NN3048  | 0.544            |
| NN3012T | 1.76             | NN3024T | 0.966            | NN3052  | 0.526            |
| NN3013T | 1.64             | NN3026T | 0.921            | NN3056  | 0.492            |
| NN3014T | 1.53             | NN3028  | 0.861            | NN3060  | 0.474            |
| NN3015T | 1.47             | NN3030  | 0.816            | NN3064  | 0.444            |

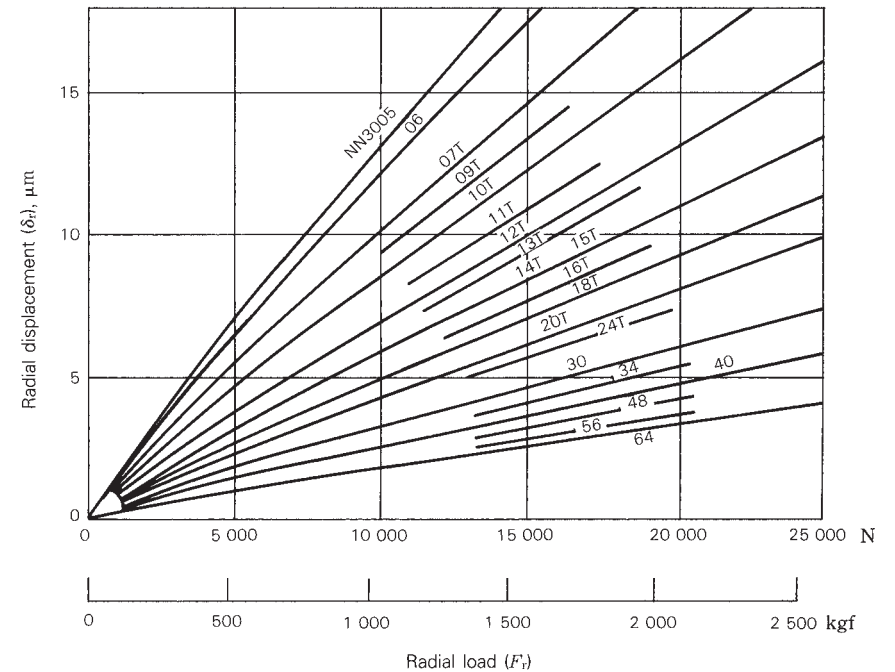


Fig. 1

### 5.8 Misalignment, maximum rolling-element load and moment for deep groove ball bearings

#### 5.8.1 Misalignment angle of rings and maximum rolling-element load

There are occasions when the inner and outer rings of deep groove bearings are forced to rotate out of parallel, whether from shaft deflection or mounting error. The allowable misalignment can be determined from the relation between the inner or outer ring deflection angle  $\theta$  and maximum rolling-element load  $Q_{max}$ .

For standard groove radii, the relation between  $\theta$  and  $Q_{max}$  (see Fig. 1) is given by Equation (1).

$$Q_{max} = K D_w^2 \left\{ \sqrt{\left( \frac{R_i}{2m_0} \theta \right)^2 + \cos^2 \alpha_0} - 1 \right\}^{3/2} \quad (1)$$

(N), {kgf} .....

where,  $K$ : Constant determined by bearing material and design  
 Approximately for deep groove ball bearing

$$K = 717 \text{ (N-unit)}$$

$$K = 72.7 \text{ (kgf-unit)}$$

$Q_{max}$ : Maximum rolling element load (N), {kgf}

$D_w$ : Ball diameter (mm)

$R_i$ : Distance between bearing center and inner ring raceway curvature center (mm)

$$m_0 = m_0 = r_i + r_e - D_w$$

$r_i$  and  $r_e$  are inner and outer ring groove radii, respectively

$\theta$ : Inner and outer ring misalignment angle (rad)

$\alpha_0$ : Initial contact angle ( $^\circ$ )

$$\cos \alpha_0 = 1 - \frac{\Delta_r}{2m_0}$$

$\Delta_r$ : Radial clearance (mm)

When a radial load  $F_r$  equivalent to the basic static load rating  $C_{0r}=17\,800 \text{ N}$  {1 820 kgf} or basic dynamic load rating  $C_r=29\,100 \text{ N}$  {2 970 kgf} is applied on a bearing,  $Q_{max}$  becomes as follows by Equation 8 in Section 5.1.

$$F_r = C_{0r} \quad Q_{max} = 9\,915 \text{ N} \{1\,011 \text{ kgf}\}$$

$$F_r = C_r \quad Q_{max} = 16\,167 \text{ N} \{1\,650 \text{ kgf}\}$$

Since the allowable misalignment  $\theta$  during operation will vary depending on the load, it is impossible to make an unqualified statement, but if we reasonably assume  $Q_{max}=2\,000 \text{ N}$  {204 kgf}, 20% of  $Q_{max}$  when  $F_r=C_{0r}$ , we can determine from Fig. 2 that  $\theta$  will be:

$$\Delta_r = 0 \quad \theta = 18'$$

$$\Delta_r = 0.050 \text{ mm} \quad \theta = 24.5'$$

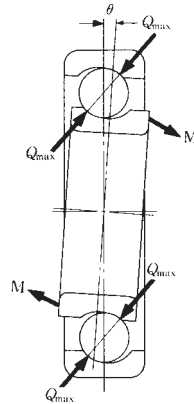


Fig. 1

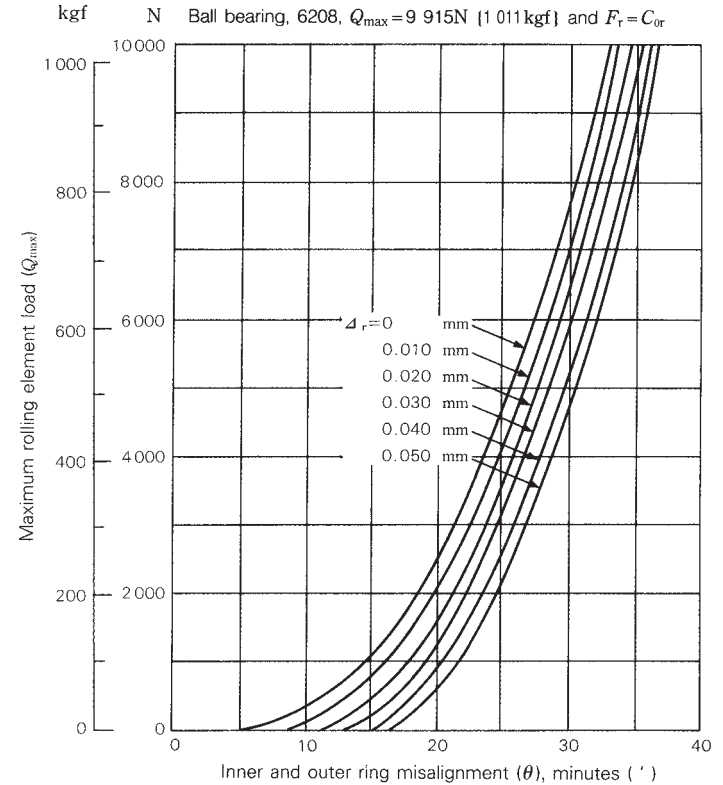


Fig. 2 Inner and outer ring misalignment and maximum rolling element load

Fig. 2 shows the relationship between  $\theta$  and  $Q_{max}$  for a 6208 ball bearing with various radial clearances  $\Delta_r$ .

**5.8.2 Misalignment of inner and outer rings and moment**

To determine the angle  $\psi$  (Fig. 3) between the positions of the ball and ball under maximum rolling-element load, for standard race way radii, Equation (2) for rolling-element load  $Q(\psi)$  can be used like Equation (1) (Page 134).

$$Q(\psi) = K D_w^2 \left\{ \sqrt{\left(\frac{R_i - \theta}{m_0}\right)^2 \cos^2 \psi + \cos^2 \alpha_0} - 1 \right\}^{3/2}$$

(N), {kgf} ..... (2)

The moment  $M(\psi)$  caused by the relative inner and outer ring misalignment from this  $Q(\psi)$  is given by,

$$M(\psi) = \frac{D_{pw}}{2} \cos \psi Q(\psi) \sin \alpha(\psi)$$

where,  $D_{pw}$ : Ball pitch diameter (mm)  
 $\alpha(\psi)$ , as used here, represents the local rolling element contact angle at the  $\psi$  position. It is given by,

$$\sin \alpha(\psi) = \frac{\left(\frac{R_i - \theta}{m_0}\right) \cos \psi}{\sqrt{\left(\frac{R_i - \theta}{m_0}\right)^2 \cos^2 \psi + \cos^2 \alpha_0}}$$

It is better to consider that the moment  $M$  originating from bearing can be replaced with the total moment originating from individual rolling element loads. The relation between the inner and outer ring misalignment angle  $\theta$  and moment  $M$  is as shown by Equation (3):

$$M = \sum_{\psi=0}^{2\pi} \frac{D_{pw}}{2} \cos \psi Q(\psi) \sin \alpha(\psi)$$

$$= \frac{K D_{pw} D_w^2}{2} \sum \frac{\left\{ \sqrt{\left(\frac{R_i - \theta}{m_0}\right)^2 \cos^2 \psi + \cos^2 \alpha_0} - 1 \right\}^{3/2} \left(\frac{R_i - \theta}{m_0}\right) \cos^2 \psi}{\sqrt{\left(\frac{R_i - \theta}{m_0}\right)^2 \cos^2 \psi + \cos^2 \alpha_0}}$$

(mN·m), {kgf·mm} ..... (3)

where,  $K$ : Constant determined by bearing material and design

Figs. 4 shows the calculated results for a 6208 deep groove ball bearing with various internal clearances. The allowable moment for a 6208 bearing with a maximum rolling element load  $Q_{max}$  of 2 000 N {204 kgf}, can be estimated using Fig. 2 (Page 135):

Radial clearance  $\Delta_r=0$ ,  $\theta=18'$   
 $M=60 \text{ N}\cdot\text{m}$  {6.2 kgf·mm}

Radial clearance  $\Delta_r=0.050 \text{ mm}$   $\theta=24.5'$   
 $M=70 \text{ N}\cdot\text{m}$  {7.1 kgf·mm}

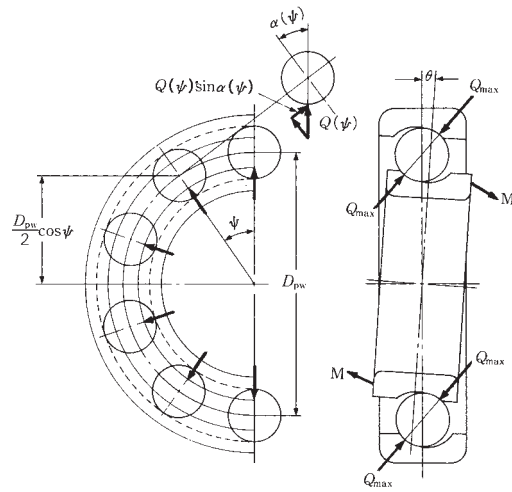


Fig. 3

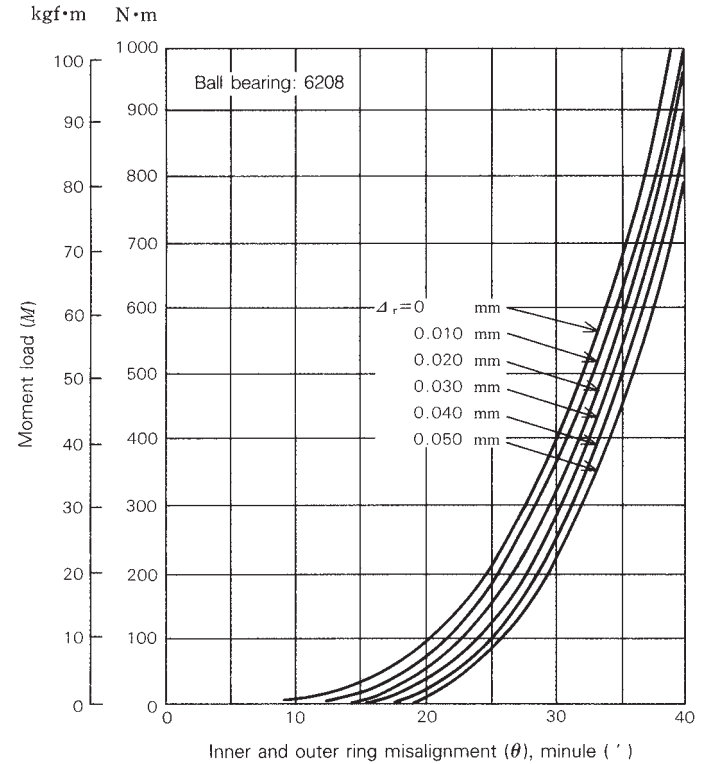


Fig. 4 Inner and outer ring misalignment and moment

### 5.9 Load distribution of single-direction thrust bearing due to eccentric load

When a pure axial load  $F_a$  is applied on a single-direction thrust bearing with a contact angle of  $\alpha=90^\circ$ , each rolling element is subjected to a uniform load  $Q$ :

$$Q = \frac{F_a}{Z}$$

where,  $Z$ : Number of rolling elements

Fig. 1 shows the distribution with any eccentric load  $F_a$  applied on a single-direction thrust bearing with a contact angle  $\alpha=90^\circ$ .

Based on Fig. 1, the following equations can be derived to determine the total elastic deformation  $\delta_{\max}$  of the rolling element under the maximum load and the elastic deformation of any other rolling element  $\delta(\psi)$ .

$$\delta_{\max} = \delta_T + \frac{\theta D_{pw}}{2} \dots \dots \dots (1)$$

$$\delta(\psi) = \delta_T + \frac{\theta D_{pw}}{2} \cos\psi \dots \dots \dots (2)$$

From Equation (1) and (2) we obtain,

$$\delta(\psi) = \delta_{\max} \left\{ 1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right\} \dots \dots \dots (3)$$

where,

$$\varepsilon = \frac{1}{2} \left( 1 + \frac{2\delta_T}{\theta D_{pw}} \right) \dots \dots \dots (4)$$

The load  $Q(\psi)$  on any rolling element is proportional to the elastic deformation  $\delta(\psi)$  of the contact surface to the  $t$  power. Thus when  $\psi=0$ , with  $Q_{\max}$  representing the maximum rolling element load and  $\delta_{\max}$  the elastic deformation, we obtain.

$$\frac{Q(\psi)}{Q_{\max}} = \left\{ \frac{\delta(\psi)}{\delta_{\max}} \right\}^t \dots \dots \dots (5)$$

$t=1.5$  (point contact),  $t=1.1$  (line contact)

From Equations (3) and (5), we obtain,

$$\frac{Q(\psi)}{Q_{\max}} = 1 - \left\{ \frac{1}{2\varepsilon} (1 - \cos\psi) \right\}^t \dots \dots \dots (6)$$

Since the eccentric load  $F_a$  acting on a bearing must be the sum of the individual rolling element loads, we obtain ( $Z$  is the number of the rolling elements),

$$\begin{aligned} F_a &= \sum_{\psi=0}^{2\pi} Q(\psi) \\ &= \sum_{\psi=0}^{2\pi} Q_{\max} \left\{ 1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right\}^t \\ &= Q_{\max} Z J_A \dots \dots \dots (7) \end{aligned}$$

Based on Fig. 1, the moment  $M$  acting on the shaft with  $\psi=90^\circ$  as the axis is,

$$\begin{aligned} M &= \sum_{\psi=0}^{2\pi} Q(\psi) \frac{D_{pw}}{2} \cos\psi \\ &= \sum_{\psi=0}^{2\pi} Q_{\max} \frac{D_{pw}}{2} \left\{ 1 - \frac{1}{2\varepsilon} (1 - \cos\psi) \right\}^t \cos\psi \\ &= Q_{\max} Z \frac{D_{pw}}{2} J_R \dots \dots \dots (8) \end{aligned}$$

Values for  $\varepsilon$  and the corresponding  $J_A$  and  $J_R$  values for point contact and line contact from Equations (7) and (8) are listed in Table 1.

#### Sample Calculation

Find the maximum rolling element load for a 51130X single-direction thrust ball bearing ( $\phi 150 \times \phi 190 \times 31$  mm) that sustains an axial load of 10 000 N {1 020 kgf} at a position 80 mm out from the bearing center.

$$e=80, D_{pw} \doteq \frac{1}{2} (150+190)=170$$

$$\frac{2e}{D_{pw}} = \frac{2 \times 80}{170} = 0.941$$

$$Z=32$$

Using Table 1, the value for  $J_A$  corresponding to  $2e/D_{pw}=0.941$  is 0.157. Substituting these values into Equation (7), we obtain,

$$\begin{aligned} Q_{\max} &= \frac{F_a}{Z J_A} = \frac{10\,000}{32 \times 0.157} = 1\,990 \text{ (N)} \\ &= \frac{1\,020}{32 \times 0.157} = 203 \text{ (kgf)} \end{aligned}$$

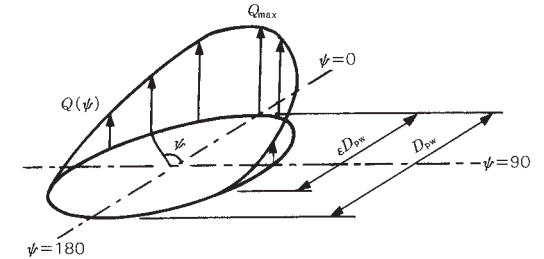
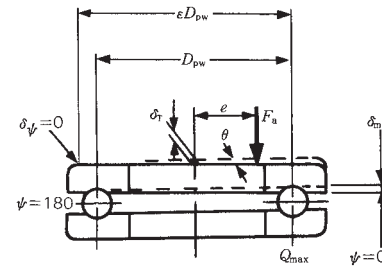


Fig. 1

Table 1  $J_R$  and  $J_A$  values for single-direction thrust bearings

| $\varepsilon$ | Point contact                     |        |        | Line contact                      |        |        |
|---------------|-----------------------------------|--------|--------|-----------------------------------|--------|--------|
|               | $\frac{2e/D_{pw}}{2M/D_{pw} F_a}$ | $J_R$  | $J_A$  | $\frac{2e/D_{pw}}{2M/D_{pw} F_a}$ | $J_R$  | $J_A$  |
| 0             | 1.0000                            | 1/Z    | 1/Z    | 1.0000                            | 1/Z    | 1/Z    |
| 0.1           | 0.9663                            | 0.1156 | 0.1196 | 0.9613                            | 0.1268 | 0.1319 |
| 0.2           | 0.9318                            | 0.1590 | 0.1707 | 0.9215                            | 0.1737 | 0.1885 |
| 0.3           | 0.8964                            | 0.1892 | 0.2110 | 0.8805                            | 0.2055 | 0.2334 |
| 0.4           | 0.8601                            | 0.2117 | 0.2462 | 0.8380                            | 0.2286 | 0.2728 |
| 0.5           | 0.8225                            | 0.2288 | 0.2782 | 0.7939                            | 0.2453 | 0.3090 |
| 0.6           | 0.7835                            | 0.2416 | 0.3084 | 0.7480                            | 0.2568 | 0.3433 |
| 0.7           | 0.7427                            | 0.2505 | 0.3374 | 0.6999                            | 0.2636 | 0.3766 |
| 0.8           | 0.6995                            | 0.2559 | 0.3658 | 0.6486                            | 0.2658 | 0.4098 |
| 0.9           | 0.6529                            | 0.2576 | 0.3945 | 0.5920                            | 0.2628 | 0.4439 |
| 1.0           | 0.6000                            | 0.2546 | 0.4244 | 0.5238                            | 0.2523 | 0.4817 |
| 1.25          | 0.4338                            | 0.2289 | 0.5044 | 0.3598                            | 0.2078 | 0.5775 |
| 1.67          | 0.3088                            | 0.1871 | 0.6060 | 0.2340                            | 0.1589 | 0.6790 |
| 2.5           | 0.1850                            | 0.1339 | 0.7240 | 0.1372                            | 0.1075 | 0.7837 |
| 5.0           | 0.0831                            | 0.0711 | 0.8558 | 0.0611                            | 0.0544 | 0.8909 |
| $\infty$      | 0                                 | 0      | 1.0000 | 0                                 | 0      | 1.0000 |

$e$ : Distance between bearing center and loading point  
 $D_{pw}$ : Rolling-element pitch diameter