11. Load calculation of gears

11.1 Calculation of loads on spur, helical, and double-helical gears

There is an extremely close relationship among the two mechanical elements, gears and rolling bearings. Gear units, which are widely used in machines, are almost always used with bearings. Rating life calculation and selection of bearings to be used in gear units are based on the load at the gear meshing point.

The load at the gear meshing point is calculated as follows:

Spur gear:

$$P_1=P_2=\frac{9\ 550\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{9\ 550\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots (N)$$

$$=\frac{974\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{974\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots \{kgf\}$$

$$S_1=S_2=P_1\tan\alpha$$

The magnitudes of the forces P_2 and S_2 applied to the driven gear are the same as P_1 and S_1 respectively, but the direction is opposite.

Helical gear:

$$P_1=P_2=\frac{9\ 550\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{9\ 550\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots (N)$$

$$=\frac{974\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{974\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots \{kgf\}$$

$$S_1=S_2=\frac{P_1\tan\alpha_n}{\cos\beta}$$

$$T_1=T_2=P_1\tan\beta$$

The magnitudes of the forces P_2, S_2, and T_2 applied to the driven gear are the same as P_1, S_1, and T_1 respectively, but the direction is opposite.

Double-helical gear:

$$P_1=P_2=\frac{9\ 550\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{9\ 550\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots (N)$$

$$=\frac{974\ 000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{974\ 000H}{n_2\left(\frac{d_{p2}}{2}\right)} \cdots\cdots \{kgf\}$$

$$S_1=S_2=\frac{P_1\tan\alpha_n}{\cos\beta}$$

where, P: Tangential force (N), {kgf}
S: Separating force (N), {kgf}
T: Thrust (N), {kgf}
H: Transmitted power (kW)
n: Speed (min^{-1})
d_p: Pitch diameter (mm)
α: Gear pressure angle
α_n: Gear normal pressure angle
β: Twist angle
Subscript 1: Driving gear
Subscript 2: Driving gear

In the case of double-helical gears, thrust of the helical gears offsets each other and thus only tangential and separating forces act. For the directions of tangential, separating, and thrust forces, please refer to Figs. 1 and 2.

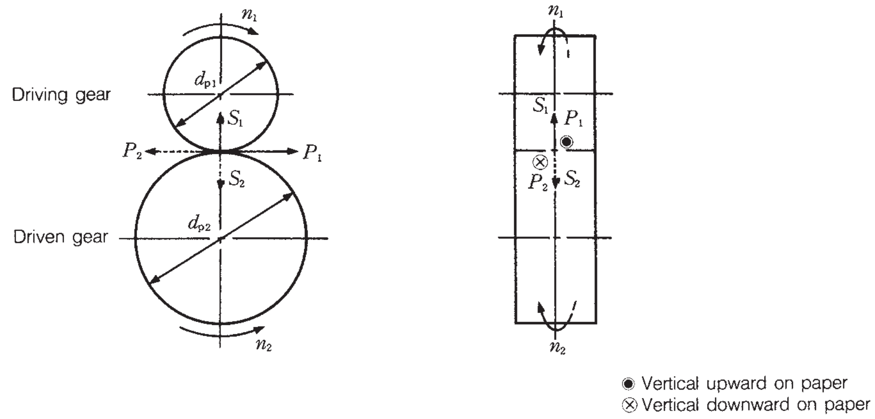


Fig. 1 Spur gear

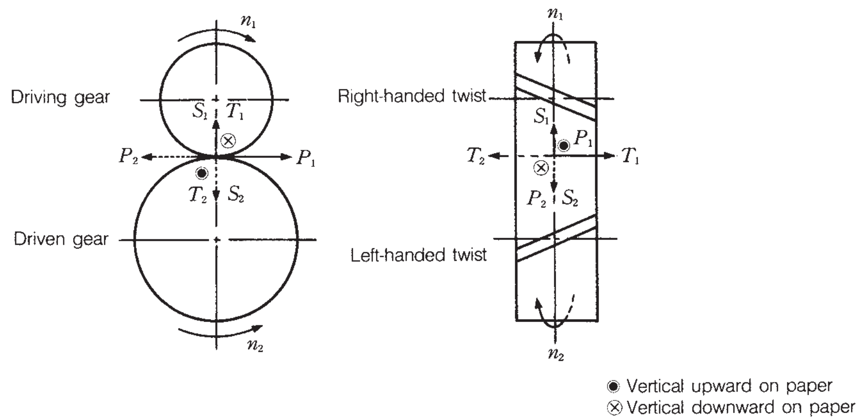


Fig. 2 Helical gear

The thrust direction of the helical gear varies depending on the gear running direction, gear twist direction, and whether the gear is driving or driven.
The directions are as follows:

The force on the bearing is determined as follows:

Tangential force:

$$P_1=P_2=\frac{9\,550\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{9\,550\,000H}{n_2\left(\frac{d_{p2}}{2}\right)} \quad \cdots\cdots\cdots \text{(N)}$$

$$=\frac{974\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}=\frac{974\,000H}{n_2\left(\frac{d_{p2}}{2}\right)} \quad \cdots\cdots \{\text{kgf}\}$$

Separating force: $S_1=S_2=P_1\frac{\tan\alpha_n}{\cos\beta}$

Thrust: $T_1=T_2=P_1\cdot\tan\beta$

The same method can be applied to bearings C and D.

Table 1

Load classification		Bearing A		Bearing B	
Radial load	From P_1	$P_A=\frac{b}{a+b}P_1$	⊗	$P_B=\frac{a}{a+b}P_1$	⊗
	From S_1	$S_A=\frac{b}{a+b}S_1$	⬆	$S_B=\frac{a}{a+b}S_1$	⬆
	From T_1	$U_A=\frac{d_{p1}/2}{a+b}T_1$	⬆	$U_B=\frac{d_{p1}/2}{a+b}T_1$	⬇
Combined radial load		$F_{rA}=\sqrt{P_A^2+(S_A+U_A)^2}$		$F_{rB}=\sqrt{P_B^2+(S_B-U_B)^2}$	
Axial load		$F_a=T_1$ ⬅			

Load direction is shown referring to left side of Fig. 3.

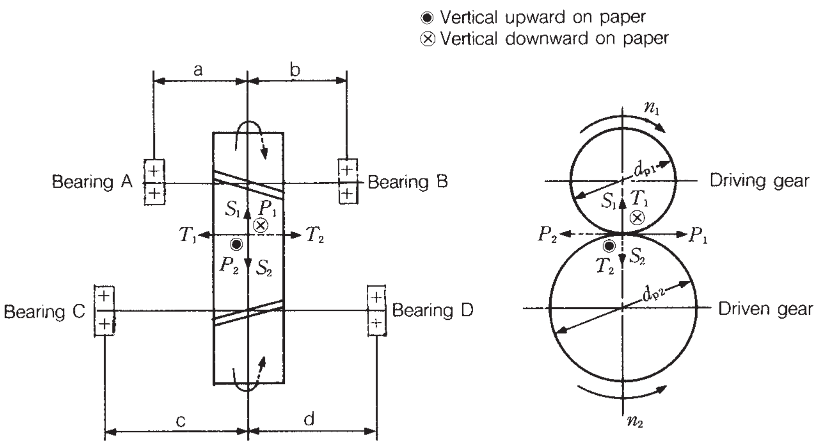


Fig. 3

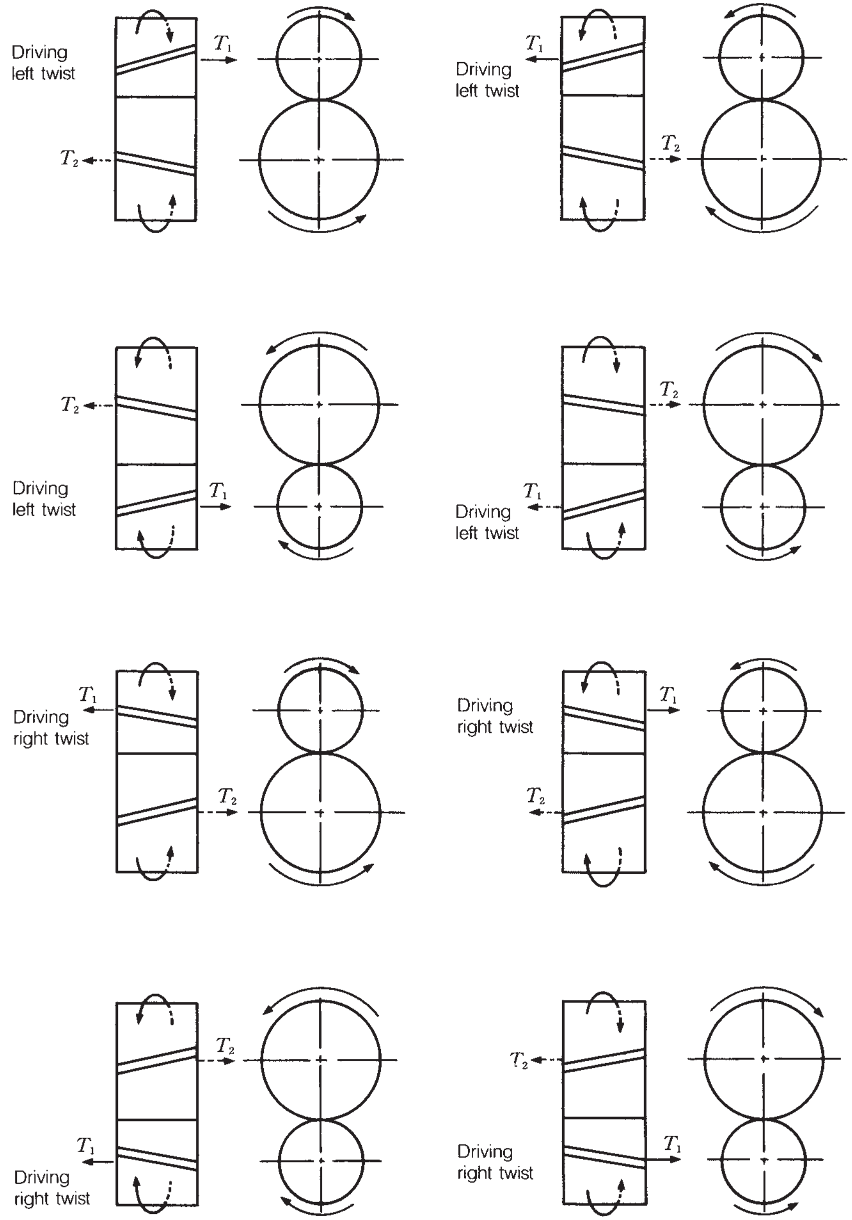


Fig. 4 Thrust direction

11.2 Calculation of load acting on straight bevel gears

The load at the meshing point of straight bevel gears is calculated as follows:

$$P_1=P_2=\frac{9\ 550\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{9\ 550\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots\ \text{(N)}$$

$$=\frac{974\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{974\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots\ \{\text{kgf}\}$$

$$D_{m1}=d_{p1}-w\ \sin\delta_1$$
$$D_{m2}=d_{p2}-w\ \sin\delta_2$$

$$S_1=P_1\tan\alpha_n\ \cos\delta_1$$
$$S_2=P_2\tan\alpha_n\ \cos\delta_2$$

$$T_1=P_1\tan\alpha_n\ \cos\delta_1$$
$$T_2=P_2\tan\alpha_n\ \cos\delta_2$$

where, D_m: Average pitch diameter (mm)
d_p: Pitch diameter (mm)
w: Gear width (pitch line length) (mm)
α_n: Gear normal pressure angle
δ: Pitch cone angle

Generally, $\delta_1+\delta_2=90°$. In this case, S_1 and T_2 (or S_2 and T_1) are the same in magnitude but opposite in direction. S/P and T/P for δ are shown in Fig. 3. The load on the bearing can be calculated as shown below.

Table 1

◉ Vertical upward on paper
⊗ Vertical downward on paper

Load classification		Bearing A	Bearing B	Bearing C	Bearing D
Radial load	From P	$P_A=\frac{b}{a}P_1$ ◉	$P_B=\frac{a+b}{a}P_1$ ⊗	$P_C=\frac{d}{c+d}P_2$ ◉	$P_D=\frac{c}{c+d}P_2$ ◉
	From S	$S_A=\frac{b}{a}S_1$ ⬇	$S_B=\frac{a+b}{a}S_1$ ⬆	$S_C=\frac{d}{c+d}S_2$ ➡	$S_D=\frac{c}{c+d}S_2$ ➡
	From T	$U_A=\frac{D_{m1}}{2\cdot a}T_1$ ⬆	$U_B=\frac{D_{m1}}{2\cdot a}T_1$ ⬇	$U_C=\frac{D_{m2}}{2(c+d)}T_2$ ⬅	$U_D=\frac{D_{m2}}{2(c+d)}T_2$ ➡
Combined radial load		$F_{rA}=\sqrt{P_A^2+(S_A-U_A)^2}$	$F_{rB}=\sqrt{P_B^2+(S_B-U_B)^2}$	$F_{rC}=\sqrt{P_C^2+(S_C-U_C)^2}$	$F_{rD}=\sqrt{P_D^2+(S_D+U_D)^2}$
Axial load		$F_a=T_1$ ⬅		$F_a=T_2$ ⬇	

Load direction is shown referring to Fig. 2.

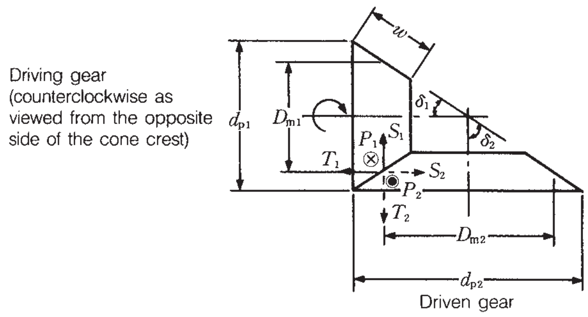


Fig. 1

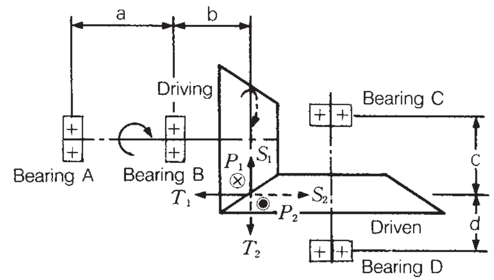


Fig. 2

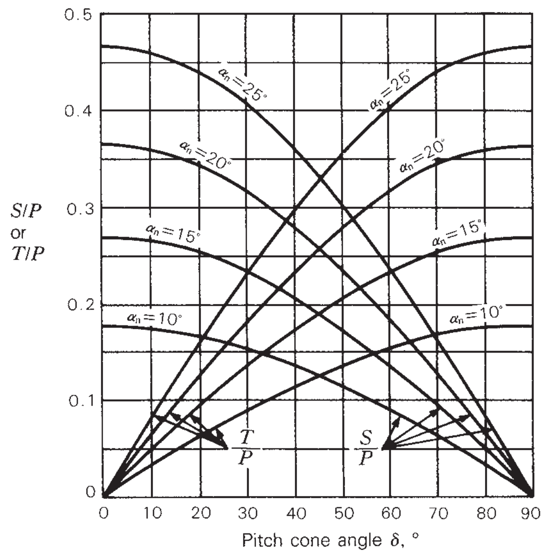


Fig. 3

11.3 Calculation of load on spiral bevel gears

In the case of spiral bevel gears, the magnitude and direction of loads at the meshing point vary depending on the running direction and gear twist direction. The running is either clockwise or counterclockwise as viewed from the side opposite of the gears (Fig. 1). The gear twist direction is classified as shown in Fig. 2. The force at the meshing point is calculated as follows:

$$P_1=P_2=\frac{9\ 550\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{9\ 550\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots \text{(N)}$$

$$=\frac{974\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{974\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots \{\text{kgf}\}$$

where, α_n: Gear normal pressure angle
β: Twisting angle
δ: Pitch cone angle
w: Gear width (mm)
D_m: Average pitch diameter (mm)
d_p: Pitch diameter (mm)

Note that the following applies:
$D_{m1}=d_{p1}-w\sin\delta_1$
$D_{m2}=d_{p2}-w\sin\delta_2$

The separating force S and T are as follows depending on the running direction and gear twist direction:

(1) Clockwise with right twisting or counterclockwise with left twisting

Driving gear
Separating force

$$S_1=\frac{P}{\cos\beta}(\tan\alpha_n\ \cos\delta_1+\sin\beta\ \sin\delta_1)$$

Thrust

$$T_1=\frac{P}{\cos\beta}(\tan\alpha_n\ \sin\delta_1-\sin\beta\ \cos\delta_1)$$

Driven gear
Separating force

$$S_2=\frac{P}{\cos\beta}(\tan\alpha_n\ \cos\delta_2-\sin\beta\ \sin\delta_2)$$

Thrust

$$T_2=\frac{P}{\cos\beta}(\tan\alpha_n\ \sin\delta_2+\sin\beta\ \cos\delta_2)$$

(2) Counterclockwise with right twist or clockwise with left twist

Driving gear
Separating force

$$S_1=\frac{P}{\cos\beta}(\tan\alpha_n\ \cos\delta_1-\sin\beta\ \sin\delta_1)$$

Thrust

$$T_1=\frac{P}{\cos\beta}(\tan\alpha_n\ \sin\delta_1+\sin\beta\ \cos\delta_1)$$

Driven gear
Separating force

$$S_2=\frac{P}{\cos\beta}(\tan\alpha_n\ \cos\delta_2+\sin\beta\ \sin\delta_2)$$

Thrust

$$T_2=\frac{P}{\cos\beta}(\tan\alpha_n\ \sin\delta_2-\sin\beta\ \cos\delta_2)$$

The positive (plus) calculation result means that the load is acting in a direction to separate the gears while a negative (minus) one means that the load is acting in a direction to bring the gears nearer.

Generally, $\delta_1+\delta_2=90°$. In this case, T_1 and S_2 (S_1 and T_2) are the same in magnitude but opposite in direction. The load on the bearing can be calculated by the same method as described in Section 11.2, "Calculation of load acting on straight bevel gears."

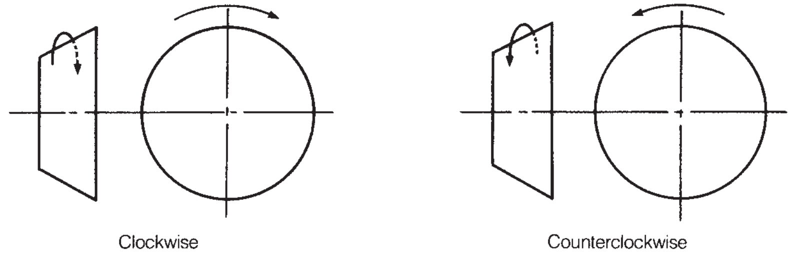


Fig. 1

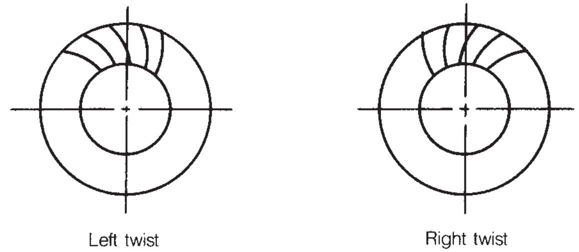


Fig. 2

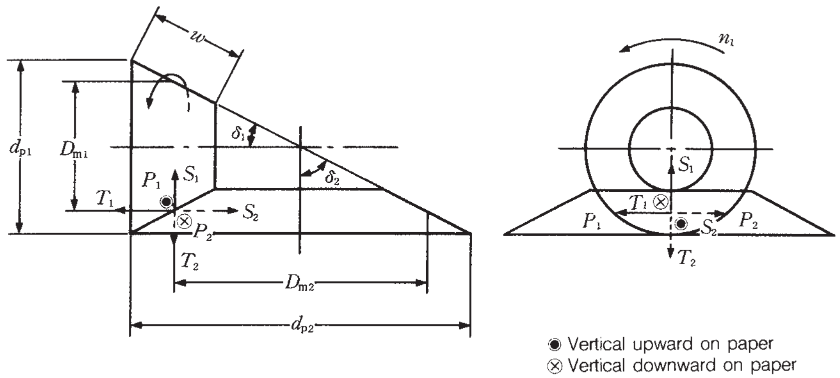


Fig. 3

11.4 Calculation of load acting on hypoid gears

The force acting at the meshing point of hypoid gears is calculated as follows:

$$P_1=\frac{9\ 550\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{\cos\beta_1}{\cos\beta_2}P_2 \quad \cdots\cdots \quad \text{(N)}$$

$$=\frac{974\ 000H}{n_1\left(\frac{D_{m1}}{2}\right)}=\frac{\cos\beta_1}{\cos\beta_2}P_2 \quad \cdots\cdots \quad \{\text{kgf}\}$$

$$P_2=\frac{9\ 550\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots \quad \text{(N)}$$

$$=\frac{974\ 000H}{n_2\left(\frac{D_{m2}}{2}\right)} \quad \cdots\cdots \quad \{\text{kgf}\}$$

$$D_{m1}=D_{m2}\frac{z_1}{z_2}\cdot\frac{\cos\beta_1}{\cos\beta_2}$$

$$D_{m2}=d_{p2}-w_2\sin\delta_2$$

where, α_n: Gear normal pressure angle
β: Twisting angle
δ: Pitch cone angle
w: Gear width (mm)
D_m: Average pitch diameter (mm)
d_p: Pitch diameter (mm)
z: Number of teeth

The separating force S and T are as follows depending on the running direction and gear twist direction:

(1) Clockwise with right twisting or counterclockwise with left twisting

Driving gear
Separating force

$$S_1=\frac{P_1}{\cos\beta_1}(\tan\alpha_n\ \cos\delta_1+\sin\beta_1\ \sin\delta_1)$$

Thrust

$$T_1=\frac{P_1}{\cos\beta_1}(\tan\alpha_n\ \sin\delta_1-\sin\beta_1\ \cos\delta_1)$$

Driven gear
Separating force

$$S_2=\frac{P_2}{\cos\beta_2}(\tan\alpha_n\ \cos\delta_2-\sin\beta_2\ \sin\delta_2)$$

Thrust

$$T_2=\frac{P_2}{\cos\beta_2}(\tan\alpha_n\ \sin\delta_2+\sin\beta_2\ \cos\delta_2)$$

(2) Counterclockwise with right twist or clockwise with left twist

Driving gear
Separating force

$$S_1=\frac{P_1}{\cos\beta_1}(\tan\alpha_n\ \cos\delta_1-\sin\beta_1\ \sin\delta_1)$$

Thrust

$$T_1=\frac{P_1}{\cos\beta_1}(\tan\alpha_n\ \sin\delta_1+\sin\beta_1\ \cos\delta_1)$$

Driven gear
Separating force

$$S_2=\frac{P_2}{\cos\beta_2}(\tan\alpha_n\ \cos\delta_2+\sin\beta_2\ \sin\delta_2)$$

Thrust

$$T_2=\frac{P_2}{\cos\beta_2}(\tan\alpha_n\ \sin\delta_2-\sin\beta_2\ \cos\delta_2)$$

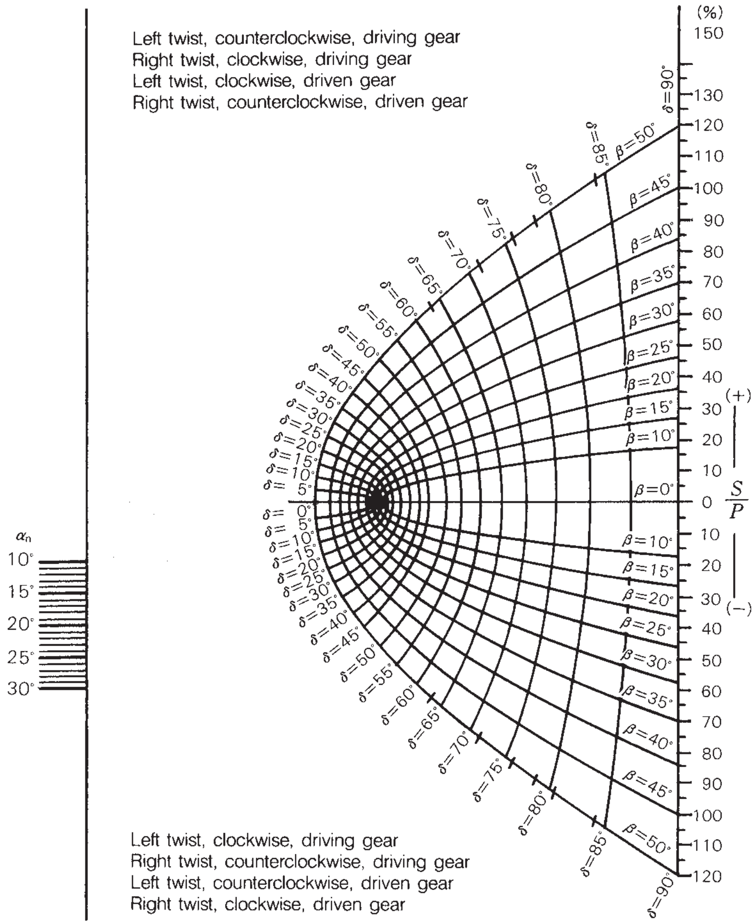
The positive (plus) calculation result means that the load is acting in a direction to separate the gears while a negative (minus) one means that the load is acting in a direction to bring the gears nearer.

For the running direction and gear twist direction, refer to Section 11.3, "Calculation of load on spiral bevel gears." The load on the bearing can be calculated by the same method as described in Section 11.2, "Calculation of load acting on straight bevel gears."

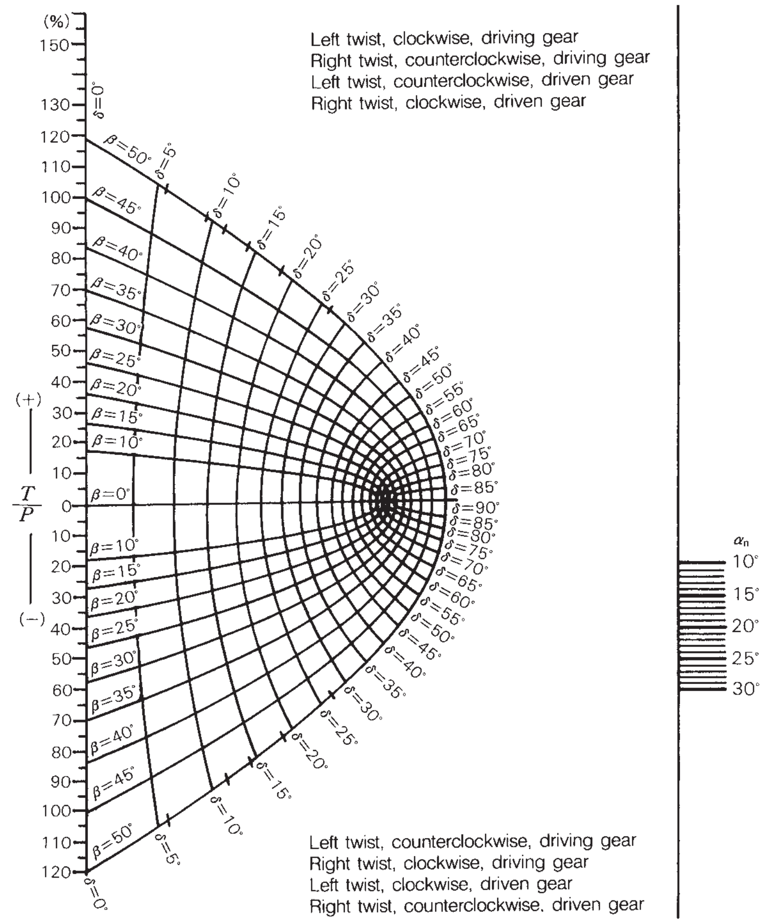
The next calculation diagram is used to determine the approximate value and direction of separating force S and thrust T.

[How To Use]
The method of determining the separating force S is shown. The thrust T can also be determined in a similar manner.

1. Take the gear normal pressure angle α_n from the vertical scale on the left side of the diagram.
2. Determine the intersection between the pitch cone angle δ and the twist angle β. Determine one point which is either above or below the β=0 line according to the rotating direction and gear twist direction.
3. Draw a line connecting the two points and read the point at which the line cuts through the right vertical scale. This reading gives the ratio (S/P, %) of the separating force S to the tangential force P in percentage.



Calculation diagram of separating force S



Calculation diagram of thrust T

11.5 Calculation of load on worm gear

A worm gear is a kind of spigot gear, which can produce a high reduction ratio with small volume. The load at a meshing point of worm gears is calculated as shown in Table 1. Symbols of Table 1 are as follows:

i: Gear ratio $\left(i=\frac{Z_2}{Z_w}\right)$

η: Worm gear efficiency $\left[\eta=\frac{\tan\gamma}{\tan(\gamma+\psi)}\right]$

γ: Advance angle $\left(\gamma=\tan^{-1}\frac{d_{p2}}{i d_{p1}}\right)$

ψ: For the frictional angle, the value obtained from $V_R=\frac{\pi d_{p1} n_1}{\cos\gamma}\times\frac{10^{-3}}{60}$ as shown in Fig. 1 is used.

When V_R is 0.2 m/s or less, then use ψ=8°.
When V_R exceeds 6 m/s, use ψ=1°4′.

α_n: Gear normal pressure angle
α_a: Shaft plane pressure angle
Z_w: No. of threads (No. of teeth of worm gear)
Z_2: No. of teeth of worm wheel
Subscript 1: For driving worm gear
Subscript 2: For driving worm gear

In a worm gear, there are four combinations of interaction at the meshing point as shown below depending on the twist directions and rotating directions of the worm gear.

The load on the bearing is obtained from the magnitude and direction of each component at the meshing point of the worm gears according to the method shown in Table 1 of Section 11.1, Calculation of loads on spur, helical, and double-helical gears.

Table 1

Force	Worm	Worm wheel
Tangential P	$\frac{9\,550\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}$ …………(N) $\frac{974\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}$ …………{kgf}	$\frac{9\,550\,000Hi\eta}{n_1\left(\frac{d_{p2}}{2}\right)}=\frac{P_1\,\eta}{\tan\gamma}=\frac{P_1}{\tan(\gamma+\psi)}$ …………(N) $\frac{974\,000Hi\eta}{n_1\left(\frac{d_{p2}}{2}\right)}=\frac{P_1\,\eta}{\tan\gamma}=\frac{P_1}{\tan(\gamma+\psi)}$ ………{kgf}
Thrust T	$\frac{9\,550\,000H\eta}{n_1\left(\frac{d_{p2}}{2}\right)}=\frac{P_1\,\eta}{\tan\gamma}=\frac{P_1}{\tan(\gamma+\psi)}$ …………(N) $\frac{974\,000H\eta}{n_1\left(\frac{d_{p2}}{2}\right)}=\frac{P_1\,\eta}{\tan\gamma}=\frac{P_1}{\tan(\gamma+\psi)}$ ………{kgf}	$\frac{9\,550\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}$ …………(N) $\frac{974\,000H}{n_1\left(\frac{d_{p1}}{2}\right)}$ …………{kgf}
Separating S	$\frac{P_1\tan\alpha_n}{\sin(\gamma+\psi)}=\frac{P_1\tan\alpha_a}{\tan(\gamma+\psi)}$ …………(N), {kgf}	$\frac{P_1\tan\alpha_n}{\sin(\gamma+\psi)}=\frac{P_1\tan\alpha_a}{\tan(\gamma+\psi)}$ …………(N), {kgf}

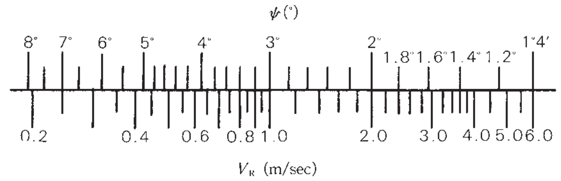


Fig. 1

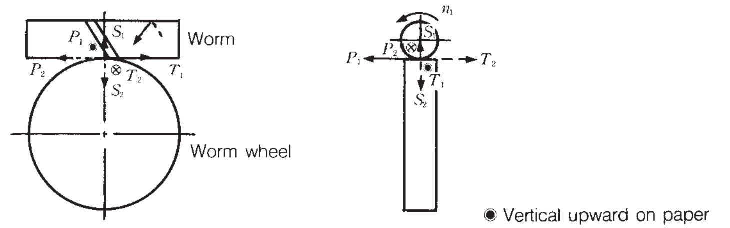


Fig. 2.1 Right twist worm gear

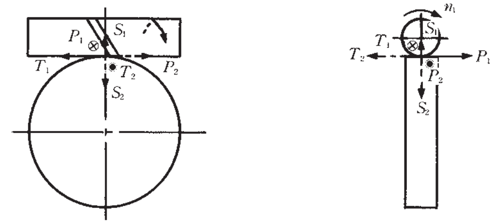


Fig. 2.2 Right twist worm gear (Worm rotation is opposite that of Fig. 2.1)

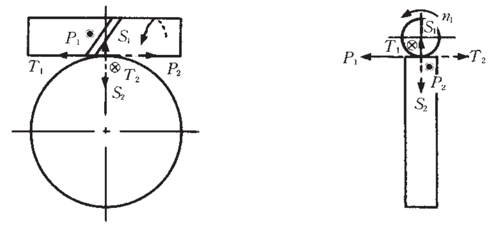


Fig. 2.3 Left twist worm gear

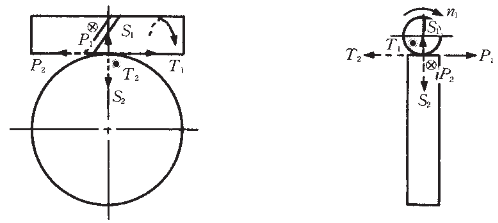


Fig. 2.4 Left twist worm gear (Worm rotation is opposite that of Fig. 2.3)